THE ENERGY SPECTRUM OF COSMIC RAY ELECTRONS BETWEEN 9
AND 300 GeV†

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Received 1978 May 26; accepted 1978 July 27

ABSTRACT

The high-energy cosmic ray electron spectrum between 9 and 300 GeV has been measured using an instrument consisting of a combination of a transition radiation detector and a shower detector. The instrument has been calibrated at accelerators over the energy range 5–300 GeV and has been exposed in a balloon flight for 9.3 m² sr hr. The design of the instrument and the data analysis procedures are described. We find that the electron spectrum is significantly steeper than the proton spectrum. If the spectrum is fitted to a single power law, a spectral index \( \alpha = 3.35 \) is obtained. This suggests the influence of radiative energy losses on galactic electrons at low energies. The results are interpreted in the context of the homogeneous model for galactic cosmic rays.

Subject headings: cosmic rays; abundances — cosmic rays; general

I. INTRODUCTION

The electron component of the cosmic rays has been of considerable interest since the first measurements of Earl (1961) and Meyer and Vogt (1961). Because of the significant role played by electromagnetic energy losses in the propagation of electrons, the information that can be obtained from a knowledge of the electron spectrum is distinctly different from that obtained from the nuclear component of the cosmic rays. Especially important are the interactions of electrons with the galactic photon and magnetic fields through Compton collisions and synchrotron radiation. Such interactions are expected to affect the shape of the electron energy spectrum by depleting the population of high-energy electrons, thus causing a steepening of the spectrum observed at Earth. A discussion of these and other relevant considerations is given in the review articles of Daniel and Stephens (1970), Ginzburg and Ptuskin (1976), and Ramaty (1974).

This paper will describe and discuss a new measurement of the cosmic ray electron spectrum above 10 GeV. Various attempts have been made in the past to measure the spectrum in this energy range using experimental techniques that generally fall into three categories: (1) electronic shower counters (Meegan and Earl 1975; Müller and Meyer 1973; Scheepmaker and Tanaka 1971; Silverberg 1976; Zatsepin 1971), (2) passive shower detectors employing emulsions (Aizu et al. 1977; Anand, Daniel, and Stephens 1973), and (3) magnetic spectrometers (Buffington, Orth, and Smoot 1975; Badhwar et al. 1977). Freier, Gilman, and Waddington (1977) have used a detector combining both electronic counters and emulsions. Unfortunately, in spite of the considerable experimental effort, no consensus has been reached among the various experiments, and large discrepancies exist both in the measured spectral shape and in the absolute flux of electrons (for a discussion, see § IV). These discrepancies have a number of possible sources: (1) The small flux of primary cosmic ray electrons makes it difficult to obtain results of good statistical accuracy. Even so, the disagreement between many of the measurements is greater than the quoted errors, indicating that statistical errors alone are not the only experimental difficulty. (2) There is a substantial background of protons which can interact to simulate electron-like events. A strong rejection against such interactions is absolutely necessary. (3) Until recently, accelerator calibrations were restricted to electron energies below 20 GeV. As a consequence, the efficiency and energy response of detectors had to be extrapolated up to much higher energies without the benefit of a direct verification at accelerators.

In view of this situation, we have built, flown, and calibrated a new cosmic ray telescope with the hope of making a more accurate and reliable measurement of the cosmic ray electron spectrum at high energies. The feature which distinguishes this detector from previous instruments is the use of a new experimental technique, transition radiation, to provide the needed discrimination against the abundant proton background. Transition radiation detectors of large cross section and light weight are easily fabricated, and thus are ideal for applications in high-altitude cosmic ray studies. Using such detectors, we have been able to develop an instrument which has good rejection against proton background while maintaining a large geometry factor to obtain good statistical accuracy. In
addition, we have taken advantage of the facilities that have recently become available at Fermilab to calibrate our instrument over the entire energy range (9–300 GeV) covered in this experiment.

This paper is organized as follows. Section II describes the basic experimental techniques used in this experiment and their incorporation in the design of the detector. Section III discusses the analysis of the flight data. Section IV presents results for the electron flux between 9 and 300 GeV and a comparison of these with the results of previous measurements. Section V gives a brief consideration of the implications of the measured electron spectrum.

The data for this experiment were gathered in a 30 hour balloon flight from Palestine, Texas, in 1975 October under 5 g cm$^{-2}$ of residual atmosphere. Accelerator tests were performed at Fermilab during 1976. The M5 beam of the Meson Laboratory was used for calibration energies of 5–50 GeV, while the Tagged Photon Laboratory was used for calibrations in the energy range 100–300 GeV. Further information is given in Table 1. Preliminary results of this experiment were reported earlier by Hartmann, Müller, and Prince (1977a, b), and this paper attempts to provide a more complete account of the techniques used, the results of the accelerator calibrations, and the analysis of the flight data.

II. TECHNIQUES AND INSTRUMENTATION

a) Techniques

An accurate measurement of the cosmic ray electron spectrum depends on the efficient identification of electrons and the strong rejection of the copious background of proton-induced events. Two separate techniques are used in this experiment to accomplish these tasks: a lead-scintillator shower counter and a transition-radiation detector.

The use of the characteristic response of a shower detector to identify electrons and measure their energy is a well known technique. Electrons interact in lead to produce showers of hard photons, electrons, and positrons. The development of this shower is measured by scintillators situated at various depths in the lead. To analyze an event, one performs a $\chi^2$ fit of the scintillator signals to the characteristic signals expected for electrons of various energies. The energy of the event is chosen to be that energy which gives the best fit to the measured signals. However, the event is a good candidate for an electron only if the $\chi^2$ parameter of the fit is sufficiently small. In practice, there is one major obstacle to the straightforward application of this technique. A proton which interacts early in the lead stack to produce a $\gamma$ will be accompanied by an electromagnetic cascade which is indistinguishable from that produced by a primary electron. Because the protons are at least 100 times more abundant than electrons in the cosmic rays, this is a serious source of background even if only a small fraction of the protons interact. The ability to further discriminate against these events is therefore highly desirable. With emissions, proton-induced events can in principle be rejected if the initial interaction is visible in the emulsion (Aizu et al. 1977; Anand, Daniel, and Stephens 1973; Frier, Gilman, and Waddington 1977). Also, deep shower detectors may be used to discriminate against proton-induced events by identifying the secondary charged mesons of a nuclear interaction that may penetrate more deeply than a typical electromagnetic cascade (Meegan and Earl 1975; Silverberg, Ormes, and Balasubrahmanyan 1973).

We have used transition radiation detectors to gain the extra needed discrimination power against proton background. The techniques used in the present experiment are based upon a series of experiments performed at the SLAC and Cornell electron accelerators which have been described previously (Cherry, Müller, and Prince 1974; Cherry et al. 1974; Prince et al. 1975). We do not wish to duplicate here the discussion given in these papers; rather we shall summarize the basic information relevant to this experiment. In general, we have found in all experiments and calibrations that the response of transition radiation detectors can be well described by the theoretical formalism of Garibian (1971) and of Ter-Mikaelian (1961, 1972). For a discussion, see Cherry (1978).

Transition-radiation X-rays are emitted when a highly relativistic particle passes through the interface between two materials having different dielectric constants. To use this effect in a practical application, an efficient "radiator" and suitable X-ray detector are needed. The radiator must have many interfaces in order to produce a significant number of transition-radiation X-rays, and it must contain only low-Z materials to avoid reabsorption of the emitted X-rays. Commercially available plastic foams make suitable radiators in many cases (Prince et al. 1975). For our purposes here, two quantities suffice to describe the

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>Experimental Specifications</th>
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<tbody>
<tr>
<td>Instrument:</td>
<td>0.45 m² sr</td>
</tr>
<tr>
<td>Geometry factor:</td>
<td>14.5 cm Dow Ethafoam (0.028 g cm⁻³)</td>
</tr>
<tr>
<td>Six radiators:</td>
<td>80% Xe, 20% CO₂, 2 cm thick</td>
</tr>
<tr>
<td>Six multiwire proportional chambers:</td>
<td>8.10 radiation lengths</td>
</tr>
<tr>
<td>Shower detector (lead and housing):</td>
<td>8.35 radiation lengths</td>
</tr>
<tr>
<td>Total detector material:</td>
<td>6.5 g cm⁻² (0.25 radiation lengths)</td>
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<tr>
<td>Matter above shower detector:</td>
<td>50–50 GeV, 5–300 GeV, 100–300 GeV</td>
</tr>
<tr>
<td>Balloon flight:</td>
<td>1975 October 7</td>
</tr>
<tr>
<td>Date:</td>
<td>National Scientific Balloon Facility, Palestine, Texas</td>
</tr>
<tr>
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<td>Float altitude:</td>
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<td>Time at float altitude:</td>
<td>Fermi National Accelerator Laboratory</td>
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<tr>
<td>Accelerator Calibrations:</td>
<td>Meson Area (M5 beam)</td>
</tr>
<tr>
<td>Location:</td>
<td>Tagged Photon Laboratory</td>
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behavior of plastic foam radiators (Cherry et al. 1974; Cherry 1978):

1. The intensity of the emitted transition radiation becomes significant only at large Lorentz factors $\gamma = E/mc^2$, and the intensity increases with particle energy up to a saturation energy corresponding to:

$$\gamma_s \approx 0.6 \frac{\omega_t}{c} (\langle l_r \rangle \langle l_p \rangle)^{1/2},$$

where $\omega_t$ is the plasma frequency of the foam material, $\langle l_r \rangle$ is the average wall thickness of the foam cells, and $\langle l_p \rangle$ is the average gap between foam cell walls.

2. At energies close to and above the saturation energy, most of the radiation is emitted near a frequency

$$\omega_{max} = \frac{\langle l_r \rangle \omega_t^2}{2 \pi c}.$$

The plastic foam used in this experiment is characterized by $\langle l_r \rangle \approx 30 \mu m$, $\langle l_p \rangle \approx 1.5 \ mm$, and $\omega_t = 21 \ eV$. Using these values, we find that the Lorentz factor of the saturation energy is $\gamma_s \approx 10^4$, corresponding to 5 GeV for electrons and 10 TeV for protons. No significant radiation is emitted by protons below 2 TeV in energy; thus, the detection of a saturated transition-radiation signal can be used to distinguish electrons from protons in the energy range covered in this experiment.

The X-ray detector must be optimized to the frequency spectrum of the emitted transition-radiation X-rays. For the plastic foam used in our detector, most of the radiation is emitted near $\omega_{max} = 11 \ keV$ (for an experimental confirmation, see Cherry 1978). Furthermore, one must consider that X-rays are emitted essentially along the trajectory of the particle, and thus the X-rays and the ionization loss of the particle will be detected simultaneously. The detector should optimize sensitivity to 10 keV X-rays relative to the response to ionization loss of a relativistic particle. Xenon-filled multiwire gas proportional chambers are suitable detectors because they combine a large X-ray absorption cross section with relatively small ionization loss signal. Since the fluctuations in the ionization loss and in the X-ray signal in a typical Xe proportional chamber are rather large, one radiator–multiwire-chamber pair alone will not suffice to discriminate between particles with and without

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**Fig. 1.** Schematic cross section of the instrument

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transition radiation. We have, therefore, employed six radiator-multiwire-chamber pairs and have analyzed the signals using a likelihood analysis procedure, the details of which will be discussed later. We now proceed to describe the design of the cosmic ray detector which employs the experimental techniques discussed in this section.

b) Instrumentation

A cross section of the detector used in this experiment is shown in Figure 1. A plastic scintillator telescope consisting of the top scintillator (T1) and three scintillators in the lead stack (T2-T4) defines a geometric factor of 0.45 m² sr. T1 and T4 are also used to make a time-of-flight measurement on all events which can be used to reject events triggered by backward-moving particles. All scintillators are made of NE 110 plastic scintillator material and are 1 cm thick. The top scintillator is viewed by six RCA 8575 photomultiplier tubes, each of the shower scintillators T2-T4 is viewed by eight RCA 4516 photomultiplier tubes, and the time-of-flight measurement is made on T4 using six additional RCA 4518 photomultiplier tubes.

The shower detector consists of scintillators T2-T4 which are at depths of 4, 6, and 8 radiation lengths, respectively (1 r.l. in lead = 6.37 g cm⁻²). The 24 photomultiplier tubes used to measure the shower signals were individually tested with nanosecond light sources for linearity up to shower signals much larger than those expected for the highest energies studied in this experiment.

The transition-radiation detector consists of two radiators, each of which is followed by a 2 cm thick multiwire gas proportional chamber filled with a mixture of 80% xenon and 20% CO₂. The plastic radiators are made of Dow Ethafoam polyethylene, and each is 14.5 cm thick. Besides acting as detectors for the transition radiation, the multiwire proportional chambers also yield holographic information. The sense wires (stainless steel, 50 μm in diameter) are spaced 1 cm apart, and groups of five adjacent wires are connected to common amplifiers. This defines a spatial resolution of 5 cm and a zenith angle resolution of 3°. All groups of five wires are individually pulse-height-analyzed by means of a track-and-hold electronic readout system. This makes it possible to determine the energy loss along the track of the particle while disregarding isolated spurious signals. After determining the track of the particle, the signals from all detector elements in the instrument can be corrected for path length dependence and spatial nonuniformities. The shower detector analysis is adjusted to account for the cosine dependence of the path of the particle through the lead stack.

All data from the instrument are read out at a rate of 20 kbits s⁻¹ for on-line analysis and storage on magnetic tape. For balloon flights, the detector was enclosed in a pressurized gondola. The total weight of the detector and gondola, including power supply, was 1542 kg.

c) Accelerator Calibrations

The calibration of the cosmic ray detector at the Fermilab accelerator is a critical part of this experiment. From the calibration of the detector, we have obtained valuable information on many aspects of detector performance, including response of the transition radiation and shower detectors as a function of energy, acceptance efficiency of the instrument for electrons of different energies, and the response of the detector to high-energy hadronic background. In this section, we will discuss the energy response of the transition radiation and shower detectors. In general, we find that the high-energy response (above 20 GeV) of the transition-radiation detector and the shower detector follows the predictions based upon measurements at lower energies. In contrast, the electron acceptance efficiency and the response of the instrument to hadronic background are extremely difficult to predict accurately on an a priori basis since these are dependent on the detailed nature of the interactions of the incident particles. They must therefore be determined empirically at an accelerator. These effects will be discussed in later sections of the paper.

Figure 2 shows the measured and predicted response of the shower detector used in this experiment. The predicted response is given by shower curves which have been derived empirically by Müller (1972) on the basis of measurements at lower energies. The agreement between measured and expected signals is very close for all energies. It should be noted from Figure 2 that the shower maximum of a 300 GeV is nearly reached in an 8 radiation length lead stack. From the accelerator calibrations, we have determined that the energy resolution of the shower detector is relatively independent of primary electron energy. The

![Figure 2: Comparison of measured and expected shower signals.](image_url)
resolution was measured to be $\sim \pm 15\%$ over the entire energy range from 9 to 300 GeV.

Figure 3 shows the average response of the transition-radiation detector as a function of energy. The results shown are from one radiator-multiwire-chamber pair. Because of the difficulty of obtaining absolute X-ray calibrations at the point of incidence of the beam, all data have been normalized to the most probable pion signal at each energy. The measured response shows the behavior predicted by theory. Between $\gamma = 10$ and $\gamma = 1000$, the detector measures the ionization loss of the particle alone, with the relativistic rise accounting for the increase in signal. At $\gamma \approx 2000$ the transition radiation begins to become significant, and above $\gamma \approx 10^4$ the transition-radiation signal saturates and remains in saturation up to the highest energies measured. As can be seen from Figure 3, all electrons above 10 GeV will emit transition radiation in saturation while protons below 2000 GeV will emit no measurable transition radiation signal. Figure 3 shows only the average response of the detector to incoming particles. The fluctuations about the average are significant, as shown by the energy loss distributions in Figure 4. Particles that do not emit transition radiation exhibit distributions which are typical Landau-Vavilov ionization loss distributions while electrons that emit transition radiation exhibit distributions that result from an ionization loss distribution folded with the X-ray transition radiation signal.

![Image of the graph](image)

**Fig. 3.**—Energy dependence of the average combined ionization and transition radiation signals for a 14.5 cm Ethafoam radiator followed by a 2 cm xenon detector ($\gamma \approx 10^4$).

![Image of the graph](image)

**Fig. 4.**—Energy loss distributions for 30 GeV protons and electrons in the transition radiation detector. Smooth curves are fits to the data from which the energy loss probability distributions $P_\gamma(x)$ and $P_e(x)$ are typically obtained (see § III).
III. DATA ANALYSIS

In this section, we discuss the analysis procedures for the flight data. First, the flight data are analyzed to obtain an uncorrected electron flux; next, corrections are applied to account for detector efficiency and atmospheric effects.

a) Track Determination

The first step in the data analysis is the use of the hodoscopic information to select those events producing a unique straight track in the instrument. The wires of chambers Ax, Bx, and Cx ("x-view") are orthogonal to those of chambers Ay, By, and Cy ("y-view"). As mentioned previously, the wires are connected in groups of five to common amplifiers. An event must fulfill the following criteria to be accepted: (1) There must be one straight track in the hodoscope, and this track must extrapolate to pass through the trigger scintillators T1 and T2. Events with an extra track which passes through T1 and T2 are rejected. (2) A maximum of four wire firings can lie along a track in each view.

Isolated wire firings or extra tracks that do not pass through the trigger scintillators do not cause the event to be rejected. These firings are considered spurious and are not used in the event analysis.

The result of the acceptance criteria is a track selection procedure which eliminates events that have ambiguous signals in the transition-radiation detector. At the same time, however, the selection procedure is relatively insensitive to isolated spurious background firings in the hodoscope. We are presently investigating variations of the track selection algorithm in order to optimize the electron acceptance efficiency while retaining good discrimination against background. The efficiency of the track selection algorithm used in the present analysis will be discussed below.

Once the angle of incidence and trajectory of the particle are known, corrections may be applied for path length variations and spatial nonuniformities of the detectors. Spatial nonuniformities were determined by mapping the response of the instrument before flight with ground-level cosmic ray muons. All amplifiers and analyzers were calibrated before flight with precision pulser, and all nonlinear characteristics were corrected for in the data. In addition, calibration data were taken during flight with protons, He particles, and nuclei heavier than boron for the purpose of monitoring any drifts in detector response with time. The data have been corrected for any such temporal variations.

b) Shower Detector Analysis

The sum of the signals measured in the three shower scintillators T2–T4 increases monotonically with the energy of the incident electron as shown in Figure 5. It is convenient to use this shower signal sum to make an initial approximation of the particle energy. The shower profile corresponding to this initial energy is then fitted to the individual signals from the shower scintillators. Let $s_i (i = 1, 2, 3)$ be the signals from the three shower scintillators, and let $t_i (i = 1, 2, 3)$ be the depths, in radiation lengths, of the three scintillators along the trajectory of the particle. Further, let $n(E, t_i)$ and $\sigma(E, t_i)$ be the average shower signals and deviations for an electron of energy $E$ at a depth of $t_i$ radiation lengths. The average predicted shower signals are shown in Figure 2. The parameter

$$\chi^2 = \frac{1}{2} \sum_{i=1}^{3} [s_i - n(E, t_i)]^2 / \sigma(E, t_i)^2$$

is then used to measure the goodness of fit of the scintillator signals to the shower profile for energy $E$. In order to account for possible fluctuations in shower development, the energy and the nominal starting point of the shower are varied until $\chi^2$ is minimized. The minimum value of $\chi^2$ is recorded, and the corresponding energy is taken to be the energy of the event. Only events with small values of $\chi^2$, typically $\chi^2 < 4$, can be due to electrons.

c) Transition Radiation Detector Analysis

The analysis of the transition-radiation detector is based on a maximum likelihood method developed during accelerator prototype tests (Cherry, Müller, and Prince 1974). Electrons produce higher signals than protons in the multiwire chambers due to the transition-radiation X-rays that they emit. The maximum likelihood method not only tests for the average size of the signals in the six chambers, but also makes use of the information contained in the distribution of the six multiwire-chamber pulse heights by testing whether or not a given set of six pulse heights is more likely to have been produced by an electron or by a background particle, presumably a proton. The probability that a particle without transition radiation produces a pulse height $x_i$ in the $i$th chamber is given by a Landau-Vavilov distribution.

FIG. 5.—Energy dependence of the sum of the signals in the three shower detector scintillators. A linear response is shown for comparison. The expected response is derived from the empirical formula of Müller (1972).
$P_{i}^{0}(x_i)$ (see Fig. 4). Clearly, the total probability that the particle produces an event with pulse heights ($x_i$, $i = 1, \ldots, 6$) in the six chambers is

$$L_p = \prod_{i=1}^{6} P_{i}^{0}(x_i).$$

Similarly, from accelerator calibrations, we can determine the probability $P_{i}^{0}(x_i)$ that an electron accompanied by transition radiation produces a pulse height $x_i$ in the $i$th chamber (see Fig. 4) and the corresponding total probability for the event,

$$L_e = \prod_{i=1}^{6} P_{i}^{0}(x_i).$$

To identify the character of a given event, we then define the “likelihood ratio,” $L = L_e/L_p$. Electrons are expected to have $L > 1$ whereas protons, which do not emit transition radiation, are expected to have $L < 1$. Likelihood distributions obtained in this fashion from accelerator and flight data will be described below (§ IIId). First we will comment on the procedures for obtaining the probability distributions for the signals of electrons and protons in the multiwire chambers.

The input probability distributions for electrons, $P_{i}^{0}(x)$, were derived from accelerator calibrations while the proton background distributions, $P_{i}^{0}(x)$, were obtained from flight data. Two considerations must be taken into account in obtaining these distributions for a likelihood analysis. First, the pulse heights produced by a particle in different chambers should be independent of each other; that is, the probability of a signal in one chamber should not depend on the magnitude of the signal in a different chamber. Second, to optimize discrimination power, the probability distributions should be adequately known. In regard to these two points, special care must be taken in deriving the proton distribution for the likelihood analysis since protons may interact in the transition-radiation detector to produce enhanced, correlated signals which can simulate transition-radiation effects in the multiwire chambers. We have

![Figure 6](image.png)

**Fig. 6.** Correlation plot of the deviation from the expected shower signal at 6 radiation lengths versus the transition-radiation likelihood ratio for a sample of singly charged flight events.
used a sample of interacting protons from the flight data which were obtained by requiring the presence of a shower, but one that does not fit well to an electron-like shower profile. Such a selection yields events that are characteristic of the most troublesome proton background.

d) Selection of Electron Events

Having determined the energy of the events, computed the likelihood parameter $L$, and calculated the $\chi^2$ parameter, we proceed to determine the number of electrons in the data sample. Events were first divided into eight energy bins. Helium particles were eliminated by requiring a singly charged particle signal in the top scintillator, T1, that was less than 2.2 times the most probable signal for protons. The effectiveness of this procedure in eliminating He particles was verified by a separate likelihood test which discriminated between electrons and He nuclei. No significant He background was found to be present.

It was determined from accelerator calibrations that 90–95% of all electrons have shower fits with $\chi^2 < 4.0$. Consequently, electrons are expected to appear as particles with $L > 1$ and $\chi^2 < 4.0$. We first illustrate this effect qualitatively in Figure 6 for a sample of flight data by showing the correlation between the likelihood ratio $L$ and the deviation from the expected shower signal in one detector. Clearly, a subset of events is well separated and distinguishes itself due to large $L$ values ($L > 1$) and small deviations from expected shower signals (indicative of small $\chi^2$ values). The discrimination of the instrument between electrons and protons is further demonstrated in Figure 7, where we plot distributions of the likelihood ratio $L$ for events in the energy ranges 9–12 GeV and 50–90 GeV that have electron-like showers, that is, $\chi^2 < 4.0$. As in Figure 6, two populations of particles are apparent: electrons with $L > 1$ that exhibit transition radiation, and protons with $L < 1$ that do not. Figure 7 (top and bottom) shows that the number of proton-induced showers exceeds the number of electron events by at least a factor of 3 even when a good fit to an electron-like shower is required. This illustrates the importance of the transition-radiation detectors in our experiment. We have compared these distributions with likelihood ratio distributions derived from accelerator calibrations for electrons and hadrons. These are shown in Figure 7 as smooth curves. The flight distributions are well explained by the superimpositions of the individual likelihood ratio distributions for electrons and hadrons found from the accelerator calibrations. On the basis of these distributions, the proton contamination in a data sample of electron-like events with good shower fit ($\chi^2 < 4$) and good transition radiation signal ($L > 1$) is on the order of a few percent at low energies and on the order of 10%, at high energies. The increase in proton background with energy is at least partially due to the relativistic rise in the ionization loss signal of protons.

The preceding analysis demonstrates the ability of the transition-radiation detectors to identify electrons and reject background protons. We now wish to confirm this discrimination power by an alternate method which estimates the proton contamination in the data more quantitatively.

We consider the set of all flight events which show a good transition-radiation signal ($L > 1$). If this sample of data is composed primarily of electron events, then the majority of events must be characterized by good shower fits. We therefore investigate the distribution of $\chi^2$ values in this data sample. From accelerator calibrations, we know that 90–95% of all electrons exhibit good shower fits ($\chi^2 < 4$), while only about 50% of all interacting protons are found with $\chi^2 < 4$. Consequently, if the sample of flight data with good

Fig. 7.—Likelihood ratio distributions for flight data. The likelihood ratio is a measure of the strength of the transition radiation signal. The smooth curves are fits to the flight data derived from accelerator calibrations.

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transition-radiation signal exhibits only a few events with poor shower fits, then the proton contamination must be small. If, on the other hand, a significant fraction of events have poor shower fits, we must suspect a sizable proton contamination.

We now proceed to make this analysis quantitative. The total number of events with good transition radiation signal is:

$$N_T = N_1 + N_2,$$

where $N_1 = \text{measured number of electron-like events with } L > 1 \text{ and } \chi^2 < 4$; and $N_2 = \text{measured number of events with } L > 1 \text{ but poor shower fit } (\chi^2 > 4)$. We wish to estimate the number of electrons and protons in the sample of electron-like events having $\chi^2 < 4$ and $L > 1$. We therefore define

$$N_1 = N_e + N_p,$$

where $N_e = \text{number of true electron events having } L > 1 \text{ and } \chi^2 < 4$, $N_p = \text{number of proton events having } L > 1 \text{ and } \chi^2 < 4$. If we know the probability, $f_e$, for an electron with $L > 1$ to exhibit a good shower fit and the probability, $f_p$, for an interacting proton with $L > 1$ to exhibit a good shower fit, we can relate $N_1$ and $N_e$ directly to the total number of events with $L > 1$:

$$N_T = f_e N_e + f_p N_p,$$

where $f_e = \text{probability of an electron to have } \chi^2 < 4$ (when $L > 1$), $f_p = \text{probability of an interacting proton to have } \chi^2 < 4$ (when $L > 1$). Solving equations (4) and (5) for the number of electrons and protons yields:

$$N_e = \frac{f_e}{f_e - f_p} (N_1 - f_p N_T),$$

$$N_p = \frac{f_p}{f_p - f_e} (N_1 - f_e N_T).$$

Thus, if we know the quantities $f_e$ and $f_p$ as functions of energy, equation (6) gives an estimate of the electron content and the proton background in the data sample of electron-like events with $\chi^2 < 4$ and $L > 1$. Figure 8 shows the quantities $f_e$ and $f_p$. The quantity $f_e$ is determined from accelerator calibrations with electrons of 5–200 GeV. The quantity $f_p$ is determined from flight data with interacting protons (Fig. 8, diamonds) and from accelerator calibrations with interacting pions (Fig. 8, squares). The data have been plotted at the average energy of the secondary shower produced by the primary hadron.

Table 2 shows the number of electrons, $N_e$, and the proton background, $N_p$, derived from the flight data along with the quantities $N_1$, $N_2$, $f_e$, and $f_p$ used in the analysis. It is seen that the estimates of the proton background made above on the basis of the expected likelihood distributions alone are essentially correct: the proton contamination in the sample of data with good transition radiation signal and good shower fit ($\chi^2 < 4$ and $L > 1$) is on the order of a few percent at 10 GeV and is typically 10–20% for energies up to 90 GeV. The background does seem to increase further at higher energies, although the statistical uncertainty in the magnitude of the background correction is rather large. At least some of this increase should be expected due to the fact that the electron spectrum is considerably steeper than that of protons (see § IV).

<table>
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<tr>
<th>Energy Interval (GeV)</th>
<th>$N_1$: Number of Events with $\chi^2 &lt; 4$</th>
<th>$N_2$: Number of Events with $\chi^2 &gt; 4$</th>
<th>$f_e$: Probability for Electrons with $\chi^2 &lt; 4$</th>
<th>$f_p$: Probability for Protons with $\chi^2 &lt; 4$</th>
<th>$N_e$: Number of Electrons with $\chi^2 &lt; 4$</th>
<th>$N_p$: Number of Protons with $\chi^2 &lt; 4$</th>
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<tbody>
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<td>8.9–11.9. . . . . . .</td>
<td>1079</td>
<td>84</td>
<td>0.95 ± 0.03</td>
<td>0.51 ± 0.05</td>
<td>546</td>
<td>57</td>
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<td>11.9–15.8. . . . . .</td>
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<td>65</td>
<td>0.95 ± 0.03</td>
<td>0.51 ± 0.05</td>
<td>466</td>
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</tr>
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<td>15.8–21.1. . . . . .</td>
<td>267</td>
<td>47</td>
<td>0.95 ± 0.03</td>
<td>0.51 ± 0.05</td>
<td>231</td>
<td>36</td>
</tr>
<tr>
<td>21.1–28.1. . . . . .</td>
<td>197</td>
<td>39</td>
<td>0.95 ± 0.03</td>
<td>0.51 ± 0.05</td>
<td>165</td>
<td>32</td>
</tr>
<tr>
<td>28.1–50.0. . . . . .</td>
<td>110</td>
<td>28</td>
<td>0.95 ± 0.03</td>
<td>0.51 ± 0.05</td>
<td>86</td>
<td>30</td>
</tr>
<tr>
<td>50.0–88.9. . . . . .</td>
<td>169</td>
<td>37</td>
<td>0.94 ± 0.03</td>
<td>0.50 ± 0.06</td>
<td>141</td>
<td>28</td>
</tr>
<tr>
<td>88.9–158. . . . . .</td>
<td>31</td>
<td>19</td>
<td>0.92 ± 0.03</td>
<td>0.49 ± 0.08</td>
<td>14</td>
<td>17</td>
</tr>
<tr>
<td>158–281. . . . . .</td>
<td>9</td>
<td>15</td>
<td>0.89 ± 0.03</td>
<td>0.46 ± 0.11</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>
e) Efficiency Corrections

The quantity \( N_e \) represents the number of electrons in the data sample restricted by four requirements: a unique straight track in the hodoscope, a singly charged particle signal in T1 (less than twice the most probable proton signal), a good fit to an electron-like shower profile (\( \chi^2 < 4 \)), and a good transition-radiation signal (\( L > 1 \)). To obtain the total number of electron events in the flight data, efficiency corrections must be applied for each restriction.

The largest efficiency correction must be applied for the track selection criteria used to identify straight tracks in the hodoscope. This correction was determined by applying the same criteria used in the flight data analysis to electron data from accelerator calibration runs. Data were taken at various energies and incident angles, and the results are shown in Figures 9 and 10. Figure 9 shows the efficiency as a function of energy for normal incidence of the beam on the detector. The energy dependence can be fitted by a logarithmic parametrization: \( f_E = 0.92 - 0.12 \ln (E + 1) \), where \( f_E \) is the efficiency and the energy \( E \) is in GeV. We have also investigated the dependence of efficiency on angle, and the results are shown in Figure 10 for six energies. The combined angular and energy dependence of the efficiency can be fitted by the following form:

\[
\begin{align*}
f_{E, \theta} &= 0.92 - 0.12 \ln (E + 1) - 0.0065 \theta, \quad (7)
\end{align*}
\]

where \( f_{E, \theta} \) is the efficiency at energy \( E \) (in GeV) and angle \( \theta \) (in degrees). The assumption of a linear dependence of efficiency on angle implies that the overall efficiency of the detector is just the efficiency measured at the average incident-particle angle. Thus, in order to determine the overall efficiency, we must know the differential angular distribution of particles accepted by the instrument. This is determined by a Monte Carlo simulation and is shown in Figure 11. To eliminate a possible uncertainty about the efficiency at large angles due to the lack of calibration data, only those particles which have incident angles less than 30° (84% of all events) are accepted in the flight data.

It should be emphasized that equation (7) is simply a convenient numerical parametrization of the dependence of the efficiency on energy and angle. At present we have no quantitative model for this dependence. Two possible mechanisms that might lead to a loss of efficiency are early bremsstrahlung interactions in the material above the shower detector and backscatter from the 8-radiation-length lead stack. In both of these cases, \( \gamma \)-rays are produced which can convert in the material of the instrument, thereby producing secondary electrons that can trigger the multiwire chambers. The presence of such spurious

---

**Fig. 9.**—Energy dependence of the track selection efficiency for normal incidence of particles on the detector. Data points are from accelerator calibrations with electrons.

**Fig. 10.**—Angular dependence of the track selection efficiency for various energies. Data points are from accelerator calibrations. The average track selection efficiency, \( f_{E, \theta} \), is shown for the energies 15, 46, and 200 GeV by bars plotted at the average angle for particle acceptance. The straight lines are calculated from the parametrization: \( f_{E, \theta} = 0.92 - 0.12 \ln (E + 1) - 0.0065 \theta \) (eq. [7]).
signals in the multiwire chambers may make it impossible to identify a unique straight track in the hodoscope and may therefore lead to a loss of efficiency. It is expected that the efficiency will decrease with increasing energy due to the increased possibility of early bremsstrahlung interactions and backscatter from the lead stack. Furthermore, the rather strong dependence of efficiency on angle might be at least partially explained by the variation of the interaction path length in the instrument as a function of angle. However, a quantitative calculation of all of the effects of bremsstrahlung and backscatter involves the detailed nature of γ-ray conversion processes as well as the detailed geometry of the instrument. To account for these would require an extensive Monte Carlo calculation which we have not attempted.

We must further consider the possibility that some part of the loss of efficiency that we observe is due to the accelerator environment and not due to interactions in the instrument itself. Such an effect could lead to a measured efficiency lower than that actually encountered in flight. The magnitude of possible accelerator background is difficult to assess. One possible source of background is backscatter from the accelerator environment in the vicinity of the shower detector. Although such a component may exist, our data indicate that the major cause of rejection of events is extra wire firings in the hodoscope which cluster near the primary particle track. These cannot be explained by backscatter from the accelerator environment since such a background would be expected to be semi-isotropic. There also exists the possibility that interactions of the electrons in the material upstream from the instrument could produce γ-rays that convert in the instrument along the particle track. Although there did exist material upstream from the detector in the accelerator beam (beam defining counters and beam pipe exit windows), this was in all cases considerably less than the material in front of the shower detector in the instrument itself. We thus find it difficult to believe that upstream material is the major source of inefficiency in our accelerator calibrations. In general, we have found no strong evidence for a significant accelerator background during our detector calibrations.

We now proceed to estimate the error involved in the determination of the average track selection efficiency. The error can be divided into two parts: (1) The error introduced by the assumption that the angular dependence is linear and the energy dependence is logarithmic, estimated to be ±0.04 in absolute efficiency (see Fig. 10), and (2) the error introduced by accelerator background in the calibration measurements, estimated to be ±0.03 in absolute efficiency. Representative errors in the average track selection efficiency, f_{TS}, are shown as bars in Figure 10, plotted at the average angle of the particle distribution. The conservative nature of the error assumptions is apparent.

Figure 12 and Table 3 give the average efficiency for track selection, f_{TS}, as a function of energy. This is essentially the function \( f_{TS} \) of equation (7) evaluated at the average angle of 18°. Also shown in Figure 12 are the other efficiency factors needed to obtain the correct electron flux: \( f_{x}, f_{L} \), and \( f_{RT} \). All of these have been determined by direct accelerator calibration of the instrument with electrons. The quantity \( f_{x} \) is the efficiency factor for the requirement of a good shower fit (\( \chi^2 < 4 \)) and is identical to the quantity \( f_{x} \) determined earlier (Fig. 8). The quantity \( f_{L} \) is the efficiency factor for the requirement of a good transition-radiation signal (\( L > 1 \)). We expect \( f_{L} \) to be approximately constant since above 5 GeV the transition radiation is in saturation. This is confirmed by the data in Figure 12. The quantity \( f_{RT} \) is the efficiency factor for the requirement of a single particle signal in the top.
Fig. 12.—Energy dependence of the efficiency factors for electron selection criteria: \( f_\text{SL} \) is the efficiency for selection of electrons with good shower fit \((x^2 < 4)\); \( f_\text{T} \) is the efficiency for the selection of singly charged particles; \( f_\text{TS} \) is the efficiency for the selection of electrons with good transition radiation signal \((L > 1)\); and \( f_\text{TS} \) is the average track selection efficiency.

scintillator \((2.2 \text{ times the most probable proton signal})\). As the energy of the electron increases, bremsstrahlung interactions in the top scintillator and in the matter above it become more significant, causing an increase in the average energy loss of electrons in the top scintillator. This in turn causes a slight decrease in the efficiency \( f_\text{TS} \).

Using the four efficiency factors \( f_{e2}, f_\text{SL}, f_\text{T}, \) and \( f_\text{TS} \) given in Figure 12 and Table 3, we may now obtain the corrected number of electron events observed during flight. This number, \( N = N_{e2} / (f_{e2} f_\text{SL} f_\text{T} f_\text{TS}) \), along with the relative error \( \sigma_{N}/N \), is given in Table 3.

\[ \boxed{ \text{f) Atmospheric Corrections} } \]

Two atmospheric effects must be taken into consideration: the energy degradation of an electron by bremsstrahlung, and the production of secondary electrons and positrons in the atmosphere.

Using the calculations of Bethe and Heitler (1934), it is found that for an interstellar electron spectrum of the form \( AE^{-\alpha} \), the observed spectrum at a depth of \( \chi \) radiation lengths of atmosphere is \( dN/dE = A(\alpha^{\text{inh}})E^{-\alpha} \). Note that the correction factor is rigorous only for a single power law spectrum. Although our results will show a spectral index that may be changing from \( \alpha = 3.0 \) to \( \alpha = 3.4 \) in the energy interval 10–100 GeV, the corresponding change in the atmospheric correction factor, \( f_\text{atm} = (\alpha^{\text{inh}}) \), is small: \( f_\text{atm} = 1.24 \) for \( \alpha = 3.0 \) and \( x = 0.13 \text{ r.l.} \), while \( f_\text{atm} = 1.28 \) for \( \alpha = 3.4 \) and \( x = 0.13 \text{ r.l.} \). We will choose \( f_\text{atm} = 1.26 \) and assume it to be energy independent.

To estimate the contribution of atmospheric secondary electrons to the electron flux, we use the calculations of Orth and Buffington (1976) for 5 g cm\(^{-2}\) of atmosphere (see Fig. 4 of that paper). The contribution is found to be small at all energies and can be neglected.

\[ \boxed{\text{g) Exposure Factor} } \]

There were two coincidence modes for electrons during flight. During 20\% of the time, a low-threshold mode accepted electrons with energies down to the geomagnetic cutoff energy. At all other times, a high-threshold mode accepted all electrons above 50 GeV. On-board scalers indicated the number of coincidence triggers and the number of events actually analyzed. From these, the dead time was determined to be \( 35.2 \pm 0.5 \text{\%} \) for the low-threshold mode and \( 1.5 \pm 0.5 \text{\%} \) for the high-threshold mode. Taking into account the relative time spent in each mode, and taking into account a maximum acceptance angle of 30\°, the resulting exposure factors are: \( 4.0 \times 10^{4} \text{ m}^2 \text{ sr s} \) for electrons below 50 GeV and \( 3.3 \times 10^{4} \text{ m}^2 \text{ sr s} \) for electrons above 50 GeV.

IV. RESULTS

The results for the differential electron flux are given in Table 4 and plotted in Figure 13. We have plotted the measured flux divided by a reference spectrum of...
the form $E^{-3.5}$. The errors associated with the data result from (1) statistics; (2) uncertainty in the determination of the efficiency factors for electrons, especially the track selection efficiency; and (3) uncertainty in the procedures for determination of background in the data. Below 30 GeV, the major contribution to the error comes from the uncertainty in track selection efficiency ($\pm 10\%$), with lesser contributions from uncertainties in estimating the other efficiency factors ($\pm 7\%$). The total error is $\pm 15\%$. The statistical errors and the error in background correction in this energy range are negligible. In the energy range 30–90 GeV, the error is again dominated by uncertainty in track selection efficiency ($\pm 15\%$), but all other sources of error have non-negligible contributions. The total error in this energy range is $\pm 22\%$. Above 90 GeV, the error is primarily due to statistics and to the uncertainty in the size of the background correction required. The background correction is itself affected by statistical uncertainties since the magnitude of the correction is determined from the distribution of $\chi^2$ values of a small number of events (see § III).

No correction has been made for the finite energy resolution of the detector. From the accelerator tests we found that this resolution was nearly constant ($\pm 15\%$ full width at half-maximum) over the entire energy range. A detector with constant energy resolution will not change the measured spectral index of a power law spectrum, although the measured flux will be slightly larger than the absolute flux (by about 5% for our detector). Another possible source of error could be a bias in the scale of the energy determination. A small shift in the energy scale could lead to a sizable error in the measured flux. For instance, a $5\%$ shift in scale can cause a $15\%$ difference in flux. We have no reason to expect such a bias in our data, but we cannot rule out absolute certainty a scale shift on the order of $5\%$ in the energy determination of our experiment.

To summarize: at low energies the errors are systematic in nature, while at high energies statistical errors dominate. The most important systematic uncertainty comes from the determination of the acceptance efficiency. Any errors in this determination could affect both the magnitude and the shape of the measured electron spectrum. In addition, any errors due to uncertainties in the energy scale or the energy resolution could also change the magnitude of the observed electron flux. Such systematic effects are expected to be small.

We wish to point out that in previous publications of preliminary results of this experiment (Hartmann, Müller, and Prince 1977a, b) only statistical errors were reported. In the present paper we have attempted to present a comprehensive and rigorous error analysis taking all possible systematic errors into account.

We now discuss the general features of the measured electron spectrum. The most important characteristic

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**TABLE 4
DIFFERENTIAL ELECTRON FLUX**

<table>
<thead>
<tr>
<th>Energy Interval (GeV)</th>
<th>$E^s$ (GeV)</th>
<th>$\Delta E^t$ (GeV)</th>
<th>$N_e$: Number of Electron Events</th>
<th>$f_{\text{exp}}$: Instrumental Exposure Factor (m$^2$ sr s)</th>
<th>$dN/dE$: Differential Electron Flux (m$^2$ sr s GeV)$^{-1}$</th>
<th>$dN/dE \times E^{3.5}$ (m$^{-2}$ sr$^{-1}$ s$^{-1}$ GeV$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.9–11.9 . . . . . . .</td>
<td>10.2</td>
<td>2.94</td>
<td>2789</td>
<td>1.26</td>
<td>$4.0 \times 10^9$ (3.0 ± 0.4) $\times 10^{-1}$</td>
<td>317 ± 44</td>
</tr>
<tr>
<td>11.9–15.8 . . . . . .</td>
<td>13.6</td>
<td>3.82</td>
<td>1557</td>
<td>1.26</td>
<td>$4.0 \times 10^9$ (1.3 ± 0.2) $\times 10^{-1}$</td>
<td>323 ± 49</td>
</tr>
<tr>
<td>15.8–21.1 . . . . . .</td>
<td>18.1</td>
<td>5.19</td>
<td>709</td>
<td>1.26</td>
<td>$4.0 \times 10^9$ (4.3 ± 0.7) $\times 10^{-2}$</td>
<td>255 ± 43</td>
</tr>
<tr>
<td>21.1–28.1 . . . . . .</td>
<td>24.1</td>
<td>6.86</td>
<td>561</td>
<td>1.26</td>
<td>$4.0 \times 10^9$ (2.6 ± 0.5) $\times 10^{-2}$</td>
<td>361 ± 69</td>
</tr>
<tr>
<td>28.1–50.0 . . . . . .</td>
<td>36.0</td>
<td>20.2</td>
<td>330</td>
<td>1.26</td>
<td>$4.0 \times 10^9$ (5.2 ± 1.2) $\times 10^{-3}$</td>
<td>240 ± 55</td>
</tr>
<tr>
<td>50.0–88.9 . . . . . .</td>
<td>64.0</td>
<td>35.9</td>
<td>703</td>
<td>1.26</td>
<td>$3.3 \times 10^8$ (7.5 ± 1.7) $\times 10^{-4}$</td>
<td>196 ± 43</td>
</tr>
<tr>
<td>88.9–158 . . . . . .</td>
<td>114</td>
<td>63.7</td>
<td>97</td>
<td>1.26</td>
<td>$3.3 \times 10^8$ (5.8 ± 4.2) $\times 10^{-5}$</td>
<td>86 ± 63</td>
</tr>
<tr>
<td>158–281 . . . . . .</td>
<td>202</td>
<td>113</td>
<td>49</td>
<td>1.26</td>
<td>$3.3 \times 10^8$ (1.7 ± 1.6) $\times 10^{-5}$</td>
<td>136 ± 128</td>
</tr>
</tbody>
</table>

* $E = \int_{E_1}^{E_2} dE E^{-3.5} \int_{E_1}^{E_2} dE E^{-3.5}; a = 3.0, E_1$ and $E_2$ are the lower and upper limits of the energy interval.

† $\Delta E = \int_{E_1}^{E_2} dE E^{-s} (E)^{-a}; a = 3.0$. 

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of the electron spectrum is the spectral slope, $\alpha$ (a is the spectral index of a power law fit, $AE^{-\alpha}$, to the electron spectrum at a given energy). The measured slope of the electron spectrum appears to be at least as steep as $\alpha = 3.0$ over the entire energy range covered in this experiment. The spectrum is considerably steeper than the proton spectrum ($\alpha_p \approx 2.65$) and strongly suggests either a source spectrum for electrons different from that of protons, or the influence of energy loss effects in the energy range 10–200 GeV. A least-squares fit of our data to a single power law would yield a spectral index of $\alpha = 3.35$. However, we do not mean to imply that a single power law fit is the best interpretation of our results. We will discuss the shape of the electron spectrum and its implications in more detail in the next section.

Figure 14 shows a comparison of our data with some of the more recent experiments that have measured the flux of cosmic ray electrons. The discrepancies mentioned in the introduction to this paper are apparent. We will not attempt to resolve these discrepancies here, but rather compare and contrast the trends of the data in individual measurements. Our results are in good agreement with the measurement of Meegan and Earl (1973) and also in agreement with the results of Aizu et al. (1977). These two measurements, as well as our own, show steep spectra with best fit spectral indices in the range $\alpha = 3.2–3.4$. The measurement of Silverberg (1976) is also in fair agreement with our results, although his measured flux is somewhat higher above 30 GeV, leading to a flatter spectrum ($\alpha = 3.10 \pm .08$). Badhwar et al. (1977) have presented preliminary results, based on a subset of their data, yielding a spectral index $\alpha \approx 3.0$. Their flux at 10 GeV is about 20%, lower than that measured in our experiment. The spectral slope seen by Buffington, Orth, and Smoot (1975) and Freier, Gilman, and Waddington (1977) below 30 GeV is also approximately $\alpha \approx 3.0$, but their measured flux is smaller by a factor of 2 than that found in our experiment.

For the sake of clarity, we have omitted from Figure 14 a number of results obtained by earlier experiments. Among these, we should mention especially the pioneering work of Anand, Daniel, and Stephens (1968, 1973), who made the first measurements of the high-energy electron spectrum. This early work, along with the experiments of Scheepmaker and Tanaka (1971) and Zatsepin (1971), indicated a flat electron spectrum with a spectral index $\alpha \approx 2.7$ in the energy range from 10 GeV to a few hundred GeV. Thus, these earlier results are in definite disagreement with the more recent results presented in Figure 14. For a discussion of the measurements of the electron spectrum from a point of view supporting a flat electron spectrum with $\alpha \approx 2.7$, we refer the reader to Anand, Daniel, and Stephens (1975). Finally, we note that although our high-energy flux is comparable to that measured in the experiment of Müller and Meyer (1973) and Müller (1973), our low-energy flux is higher. As discussed by Hartmann, Müller, and Prince (1977), this discrepancy is due to an instrumental malfunction in that earlier experiment. Because of this situation, we have not included these data in Figure 14.

It is clear from this discussion that the current status of cosmic ray electron results does not allow unambiguous conclusions to be drawn. In spite of this, a few qualitative features of the data are apparent. All of the measurements presented in Figure 14 are consistent with an electron spectrum with spectral index $\alpha \geq 3.0$ in the energy range 10–30 GeV. Thus, taken as a whole, the recent data would suggest an electron spectrum steeper than that of the proton.

![Figure 14](image_url)

**Fig. 14.** Recent measurements of the differential energy spectrum of electrons multiplied by $E^{2.0}$. The dashed curve indicates the general trend of our data.
component. However, two features of the spectrum are still to be unambiguously determined. First, the discrepancies in the measurement of the flux at 10 GeV need to be resolved. Figure 14 indicates that the results of various experiments differ by as much as a factor of 2. Second, the slope of the spectrum at high energies must be more accurately determined. Our experiment and that of Meegan and Earl (1975) suggest that the spectral slope at high energies may be as steep as $\alpha = 3.4$.

We will now proceed to an interpretation of the electron spectrum based on our own results. We believe that we have a very reliable measurement since our data are based on extensive accelerator calibrations at high energies and have been shown to be free of substantial background contamination.

V. DISCUSSION

The shape of the high-energy electron spectrum is determined by processes characteristic of both the acceleration and the propagation of cosmic rays in the Galaxy. One hopes that the interpretation of the measured spectrum will lead to new insights into some of these processes. However, because of the complexity of the cosmic ray phenomenon, we cannot expect decisive answers on the basis of the electron data alone. Rather, we must discuss our results in the context of models that have been developed to account for other observations such as the nuclear cosmic ray composition. We will show that the electron data lead to certain important constraints with regard to such models.

The simplest and most popular of current models for cosmic rays is the homogeneous or “leaky box” model. In the standard formulation of this model (see, e.g., Ginzburg and Ptuskin 1976; Ormes and Freier 1978), electrons are produced uniformly throughout the volume of the Galaxy, lose energy uniformly at a rate proportional to the square of their energy, and escape from the Galaxy with an exponential scale of lifetimes. All spatial structure of the Galaxy is ignored. Quantitatively, the electron density is characterized by the equilibrium equation:

$$\frac{N(E)}{\tau(E)} + \frac{d}{dE}[kE^2N(E)] = Q(E),$$

(8)

where $N(E)$ is the density of electrons of energy $E$, $\tau(E)$ is the energy-dependent leakage lifetime of electrons, and $Q(E)$ is the source production rate of electrons. Electrons lose energy by synchrotron radiation and inverse Compton collisions (in the Thomson limit) at the rate $dE/dt = -kE^2$ with

$$k = 10^{-16}[W_{ph} + (3/2)(H_j/8\pi)](\text{GeV s})^{-1},$$

(9)

where $W_{ph}$ is the ambient photon energy density in eV cm$^{-3}$ and $H_j$ is the root mean square magnetic field component in $\mu$gauss. Such energy losses define a radiative lifetime, $\tau_R = (kE)^{-1}$, which must be compared to the time scale for leakage from the Galaxy, $\tau(E)$. The shorter of the two time scales determines the dominant loss effect for electrons.

We assume power laws in energy for both $\tau(E)$ and $Q(E)$, namely, $\tau(E) = \tau(0)E^{-\delta}$ and $Q(E) = AE^{-\gamma}$. The power law form for $\tau(E)$ is prompted by the observation of an energy-dependent path length for primary nuclei with $0.3 < \delta < 0.6$ (see Caldwell 1977; Müllner 1977; Ormes and Freier 1978 for a discussion). With these assumptions, the solution to equation (8) is (Silverberg and Ramaty 1973; Ramaty 1974)

$$N(E) = N_0E^{-(\gamma_0+1)}\int_0^E dE' E'^{-\gamma_0} \exp \left[\frac{-(1 - E'^{-\delta})}{(1 - \delta)} \frac{E'^{-\gamma}}{E_0^{-\gamma}}\right],$$

(10)

where $N_0$ is an overall normalization factor and $E_0 = (k\tau_0)^{-1}$. $E_0$ is the characteristic energy at which the leakage lifetime of electrons is equal to the radiative lifetime of electrons. Equation (10) has the asymptotic form:

$$N(E) \propto E^{-(\gamma_0+\delta)}$$

for $E \ll [k\tau_0(1 - \delta)]^{1/(\delta-1)}$

and

$$N(E) \propto E^{-(\gamma_0+1)}$$

for $E \gg [k\tau_0(1 - \delta)]^{1/(\delta-1)}$.

It can be seen from equation (10) that, up to a normalization factor, the density of electrons depends on three free parameters: $\Gamma_0$, $\delta$, and $E_0$. $(E_0$ can be arbitrarily chosen to be 1 GeV, which defines $\tau_0$ as the lifetime at 1 GeV.) A measurement of the electron spectrum cannot be used to specify all three of the parameters in the homogeneous model simultaneously. Consequently, a number of additional assumptions must be made.

1. It is customary to assume that electrons and all primary nuclei, including protons, have the same source spectrum with spectral index $\Gamma_0$. Although this assumption is not confirmed by any direct experimental evidence, it is perhaps plausible since all major cosmic ray nuclei seem to be accelerated at the sources with the same power law spectrum (Caldwell 1977).

2. Because of the power-law energy dependence of the leakage lifetime, $\tau(E)$, the observed spectral index of the primary nuclei in the homogeneous model will be $\Gamma = \Gamma_0 + \delta$, with $\Gamma \approx 2.65$ (Caldwell 1977). From the measured nuclear composition data a value of $0.3 < \delta < 0.6$ can be inferred (Caldwell 1977; Fontes, Meyer, and Perron 1977). We note here that the existence of an energy-dependent path length for primary nuclei is well established. However, it is not certain that this effect must necessarily be due to an energy-dependent leakage lifetime. Other possibilities, such as the “nested leaky-box model” of Cowssik and Wilson (1973), ascribe the energy-dependent path length to source effects while the leakage lifetime remains energy independent (i.e., $\delta = 0$). It is therefore of interest to investigate the two cases $\delta = 0$ and $0.3 < \delta < 0.6$, and their relation to the shape of the electron spectrum.
characteristic energy for $\delta = 0.4$ is quite low, $E_c \lesssim 2$ GeV. This would indicate that radiative energy loss effects begin to dominate at very low energies. Since the spectral index at the source is $\Gamma_0 = 2.65 - \delta$, the asymptotic high-energy spectrum for the case of $\delta = 0.4$ ($\Gamma_0 + 1 = 3.25$) would be flatter than for the case of $\delta = 0$ ($\Gamma_0 + 1 = 3.65$). Unfortunately, the statistical accuracy of our data is not yet sufficient to rule out either case.

While the homogeneous model can be used to estimate the characteristic energy, $E_c = (k\tau_0)^{-1}$, it cannot be used to obtain values for $k$ and $\tau_0$ independently. However, using information from other observations, we can draw some conclusions about their approximate magnitudes. Figure 17 shows the relationship of $k$ and $\tau_0$ for various values of $E_c$. A lower limit can be placed on $k$ because of the presence of the $3K$ blackbody radiation which has a photon density of $0.25$ eV cm$^{-3}$ (eq. [9]). Starlight photons and infrared photons also contribute to Compton losses for electrons. We will place a conservative lower limit on $k$ of $0.5 \times 10^{-16}$ GeV$^{-1}$ s$^{-1}$, which corresponds to a photon energy density of $0.5$ eV cm$^{-3}$. The scale on the right-hand side of Figure 17 indicates contributions to $k$ by magnetic fields in addition to the contribution by the photon fields (see eq. [9]). Magnetic field strengths on the order of 3–5 $\mu$gauss have been assumed in the past, although some authors have suggested much higher field strengths ($\gtrsim 10$ $\mu$gauss), largely on the basis of measurements of the cosmic ray electron flux (Freier, Gilman, and Waddington 1977; Webber 1977). The leakage lifetime has been directly inferred from measurements of the Be isotopes (e.g., Garcia-Munoz, Mason, and Simpson 1977) leading to a value of $17(20, -8)$ million years. We have plotted the upper and lower limits of the lifetime from the $^{10}$Be measurements in Figure 17. We
note that these measurements have been made at low energies (80 MeV) and, therefore, directly apply to the determination of $\tau_0$ only if the leakage lifetime is energy independent ($\delta = 0$).

Using Figure 17 and the values of the characteristic energy derived from the electron data, we can now draw inferences concerning the leakage lifetime $\tau_0$ and the energy loss coefficient $k$. For the case of an energy-independent leakage lifetime, values of $E_c \approx 15-45$ GeV were found to be consistent with the data. Although a wide range of lifetimes is possible in this case, reasonable lower limits for the magnetic field strength ($\sim 3 \mu$gauss) would suggest lifetimes less than $2 \times 10^7$ years. Higher magnetic fields ($\sim 10 \mu$gauss) lead to much lower estimates of the leakage lifetime in conflict with the $^{40}$Be measurements. In contrast, if the leakage lifetime is energy dependent, much higher lifetimes are allowed. The characteristic energy in this case has been found to be $E_c < 5$ GeV, implying that the leakage lifetime at 1 GeV must be larger than $10^7$ years even for magnetic field strengths of 10 $\mu$gauss. Weaker magnetic fields ($\sim 5 \mu$gauss) would imply substantially longer lifetimes ($\tau_0 \approx 10^8$ years). We note that Meegan and Earl (1975), assuming an energy-dependent leakage lifetime, have concluded that the results of their measurement of the electron spectrum imply a long lifetime, $\tau_0 > 12$ million years. Silverberg (1976) and Freier, Gilman, and Waddington (1977), assuming an energy-independent leakage lifetime, have found their results to be consistent with shorter lifetimes, $\tau_0 \approx 2$ million years.

The homogeneous model is attractive because of its simplicity. It makes a minimum number of assumptions about the detailed nature of cosmic ray propagation, and has a minimum number of free parameters to be specified. However, the model does have some obvious limitations. No allowance is made for the spatial dependence of the production and propagation of cosmic rays, and no mechanism is postulated for the leakage loss of electrons from the confinement volume. Furthermore, the homogeneous model contains the implicit assumption that the source and confinement regions of the cosmic rays are identical. All of these assumptions are open to debate. More detailed models describing the spatial dependence of the production and propagation of electrons have been discussed by various authors (e.g., Jokipii and Meyer 1968; Berkey and Shen 1969; Bulanov and Dogiel 1975; Badhwar, Daniel, and Stephens 1977; Owens and Jokipii 1977). However, there are at present no firm experimental data which specify a unique model of this type. We will therefore omit a further discussion of these models.

VI. SUMMARY

We have described a new measurement of the energy spectrum of cosmic ray electrons. This experiment is characterized by two important features: (a) a transition radiation detector was used for the first time to improve the identification of electrons and reject proton background; and (b) the instrument was fully calibrated at accelerators over the whole energy range of concern.

The steep spectral slope shown by our data indicates either a source spectrum for electrons different from that of protons, or the influence of radiative energy losses at rather low energies, well below 50 GeV. Our data yield information about the characteristic energy at which radiative losses begin to dominate, and this information can be used to discriminate the cosmic ray leakage lifetime. If the leakage lifetime for electrons is independent of energy, our data suggest moderate characteristic energies for the onset of radiative losses, 15-45 GeV, and a lifetime equal to 10 million years for magnetic field strengths of 3 $\mu$gauss. The assumption of an energy-dependent leakage lifetime suggests much lower characteristic energies and lifetimes that could be substantially larger than 10 million years.

At the present time, our experiment cannot distinguish between the two mentioned possibilities. However, we expect that further experiments with the instrument described in this paper will significantly reduce the uncertainties in the spectral slope at high energies. This should place important constraints on many aspects of current models of cosmic ray propagation, including the question of the energy dependence of the leakage lifetime.

The author wishes to thank his faculty adviser, Professor Dietrich Müller, for his support and advice during the course of this project. The work of Dr. Gernot Hartmann during all phases of this project was indispensable and sincerely appreciated. Among the many persons who assisted on this project, the author wishes to thank in particular Mr. D. Bonasera and Mr. G. Drag for the electronic work, Mr. W. Johnson and Mr. A. Kittel for mechanical design and construction, Mrs. N. Beck, Mrs. L. Glennie, and Mr. L. Littleton for computer programming, and Dr. M. Cherry and Mr. S. Jordan for assistance during accelerator calibrations. We appreciate the services and support of the National Scientific Balloon Facility for the balloon flight, and of the Fermi National Accelerator Laboratory for the accelerator calibrations.

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