Shock Acceleration at an Interplanetary Shock: A Focused Transport Approach

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1. The Guiding center Kinetic Equation for $f(x,p_{\|},p'_{\perp},t)$

Transform position x to guiding center position x_g ($x = x_g + r_g$)
Transform transverse momentum to plasma frame $p_\perp = p'_\perp + V_E$
Smallness parameter $\delta = m/q << 1$ *(small gyroradius, large gyrofrequency)*
Keep terms to order δ^o
Average over gyrophase

$$\frac{\partial f}{\partial t} + \mathbf{V_g} \cdot \frac{\partial f}{\partial \mathbf{x_g}} + \left\langle \frac{dp_{\|}}{dt} \right\rangle_\phi \frac{\partial f}{\partial p_{\|}} + \left\langle \frac{dp'_\perp}{dt} \right\rangle_\phi \frac{\partial f}{\partial p'_\perp} = \left(\frac{\partial f}{\partial t} \right)_c$$

where

$$\mathbf{V_g} = v_{\|}\mathbf{b} + \mathbf{V_E},$$

$$\left\langle \frac{dp_{\|}}{dt} \right\rangle_\phi = qE_{\|} - \frac{1}{2}\frac{p'_\perp v'_\perp}{B}\nabla_{\|}B + m\mathbf{V_E}\cdot\frac{d\mathbf{B}}{dt},$$

$$\left\langle \frac{dp'_\perp}{dt} \right\rangle_\phi = \frac{1}{2}\frac{p_{\|}v'_\perp}{B}\nabla_{\|}B - \frac{1}{2}p'_\perp\left(\nabla\cdot\mathbf{V_E} - \mathbf{b}\cdot\nabla_{\|}\mathbf{V_E}\right)$$

Transform $(p_{\|}, p'_{\perp}) \rightarrow (\varepsilon', M)$

$$\frac{\partial f}{\partial t} + \mathbf{V}_{\mathbf{g1}} \cdot \frac{\partial f}{\partial \mathbf{x}_{\mathbf{g}}} + \frac{dM}{dt}\frac{\partial f}{\partial M} + \left\langle \frac{d\varepsilon'}{dt} \right\rangle_{\phi} \frac{\partial f}{\partial \varepsilon'} = \left(\frac{\partial f}{\partial t} \right)_{c}$$

where

$$\mathbf{V}_{\mathbf{g1}} = v_{\|}\mathbf{b} + \mathbf{V}_{\mathbf{E}},$$

$$\frac{dM}{dt} \approx 0,$$

Conservation of magnetic moment

$$\left\langle \frac{d\varepsilon'}{dt} \right\rangle_{\phi} = q\mathbf{E} \cdot \mathbf{V}_{\mathbf{g2}} + M\frac{\partial B}{\partial t},$$

$$\mathbf{V}_{\mathbf{g2}} = v_{\|}\mathbf{b} + \mathbf{V}_{\mathbf{E}} + \frac{M}{q}(\nabla \times \mathbf{b})_{\|} + \frac{M}{q}\frac{\mathbf{B} \times \nabla \mathbf{B}}{B^2} + \frac{m\mathbf{b}}{qB} \times \frac{d}{dt}(v_{\|}\mathbf{b})$$

Electric field drift

Grad-B drift

Curvature drift

2. The Focused Transport Equation for $f(x,p',\mu',t)$

To get focused transport equation, assume in guiding center kinetic equation $\boldsymbol{E} = -\boldsymbol{U} \times \boldsymbol{B}$, whereby $V_E = U_\perp$

Transform parallel momentum to moving frame $(p_{||} = p'_{||} + mU_{||})$

Transform $(p'_{||}, p'_\perp) \to (p', \mu')$

$$\frac{\partial f}{\partial t} + (U_i + v\mu b_i)\frac{\partial f}{\partial x_i}$$

Convection

$$+ \frac{1}{2}(1-\mu^2)\left[v\frac{\partial b_i}{\partial x_i} + \mu\frac{\partial U_i}{\partial x_i} - 3\mu b_i b_j \frac{\partial U_i}{\partial x_j} - \frac{2}{v} b_i \frac{dU_i}{dt} + \frac{2q}{p} E_i b_i\right]\frac{\partial f}{\partial \mu}$$

Mirroring/focusing

$$+ \left[-\frac{1}{2}(1-\mu^2)\frac{\partial U_i}{\partial x_i} + \frac{1}{2}(1-3\mu^2) b_i b_j \frac{\partial U_i}{\partial x_j} - \frac{\mu}{v} b_i \frac{dU_i}{dt} + \mu\frac{q}{p} E_i b_i\right] p \frac{\partial f}{\partial p}$$

Adiabatic energy changes

$$= \frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu}\right) + Q_{PUI}$$

Parallel Diffusion

Advantages

Same mechanisms as standard cosmic-ray transport equation

No restriction on pitch-angle anisotropy - injection

Describe both (i) 1st order Fermi, and (ii) shock drift acceleration with or without scattering

Disadvantages

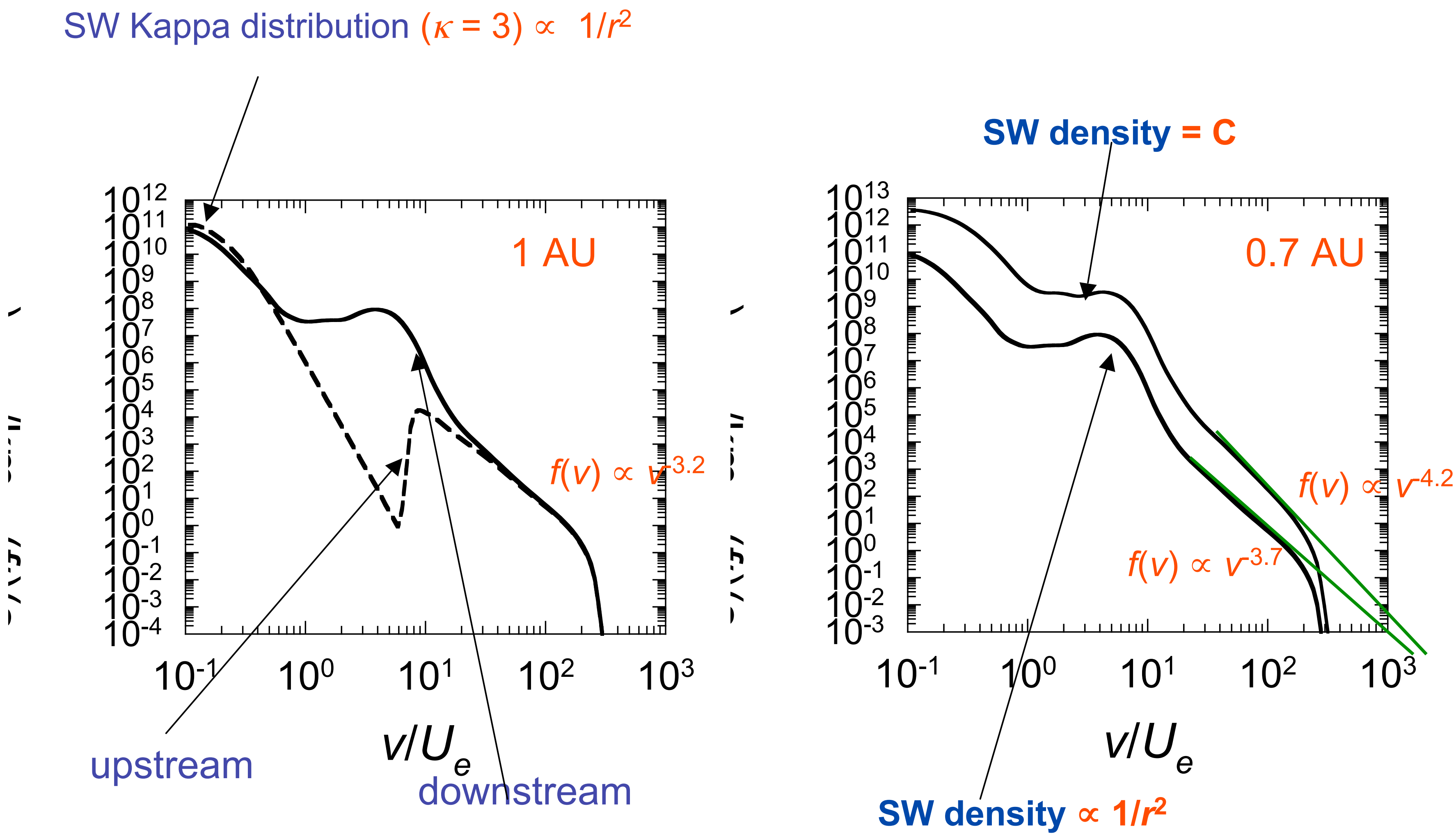
No cross-field diffusion

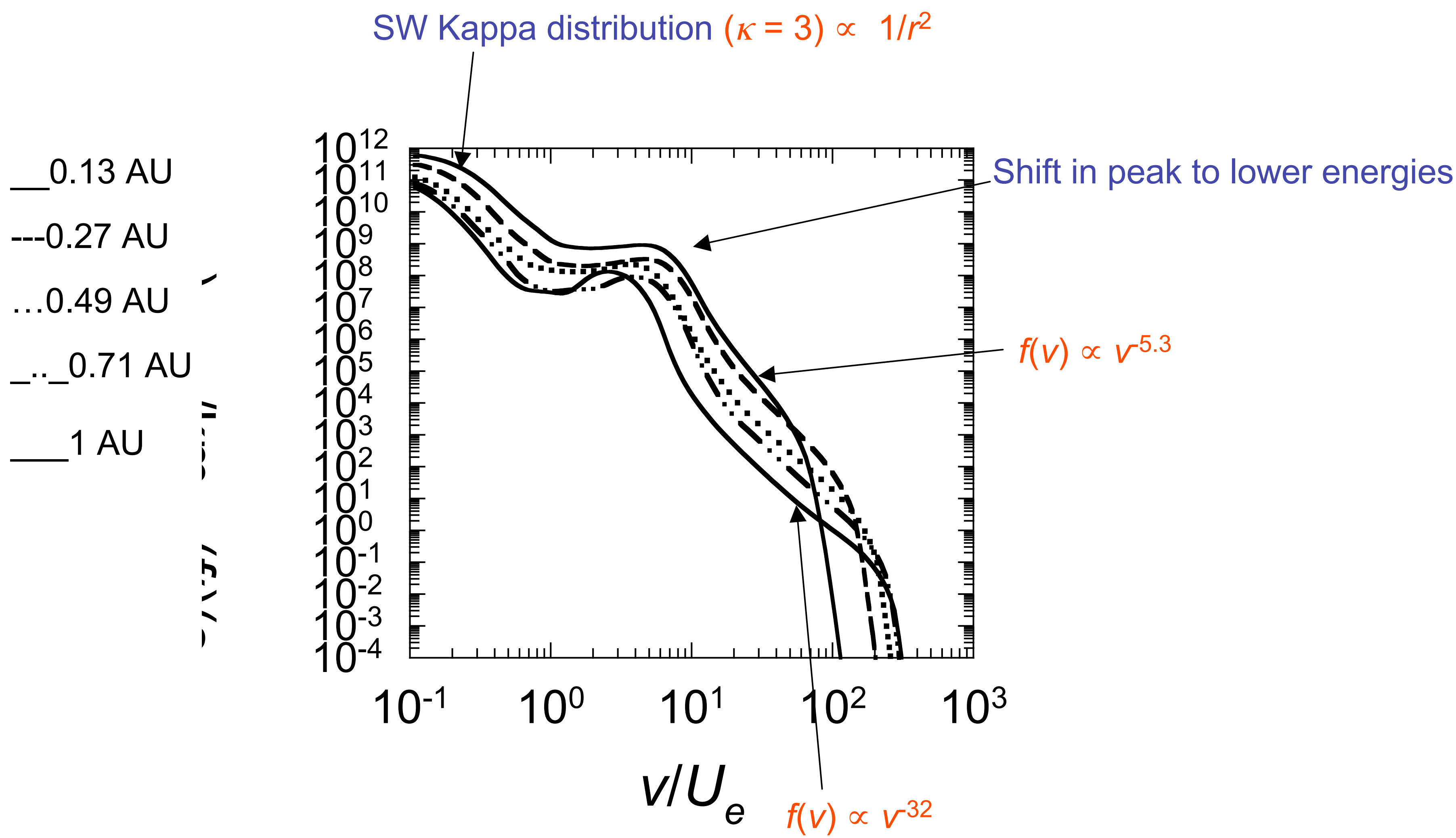
Gradient and curvature drift effect on spatial convection ignored

Magnetic moment conservation at shocks

3. Results:

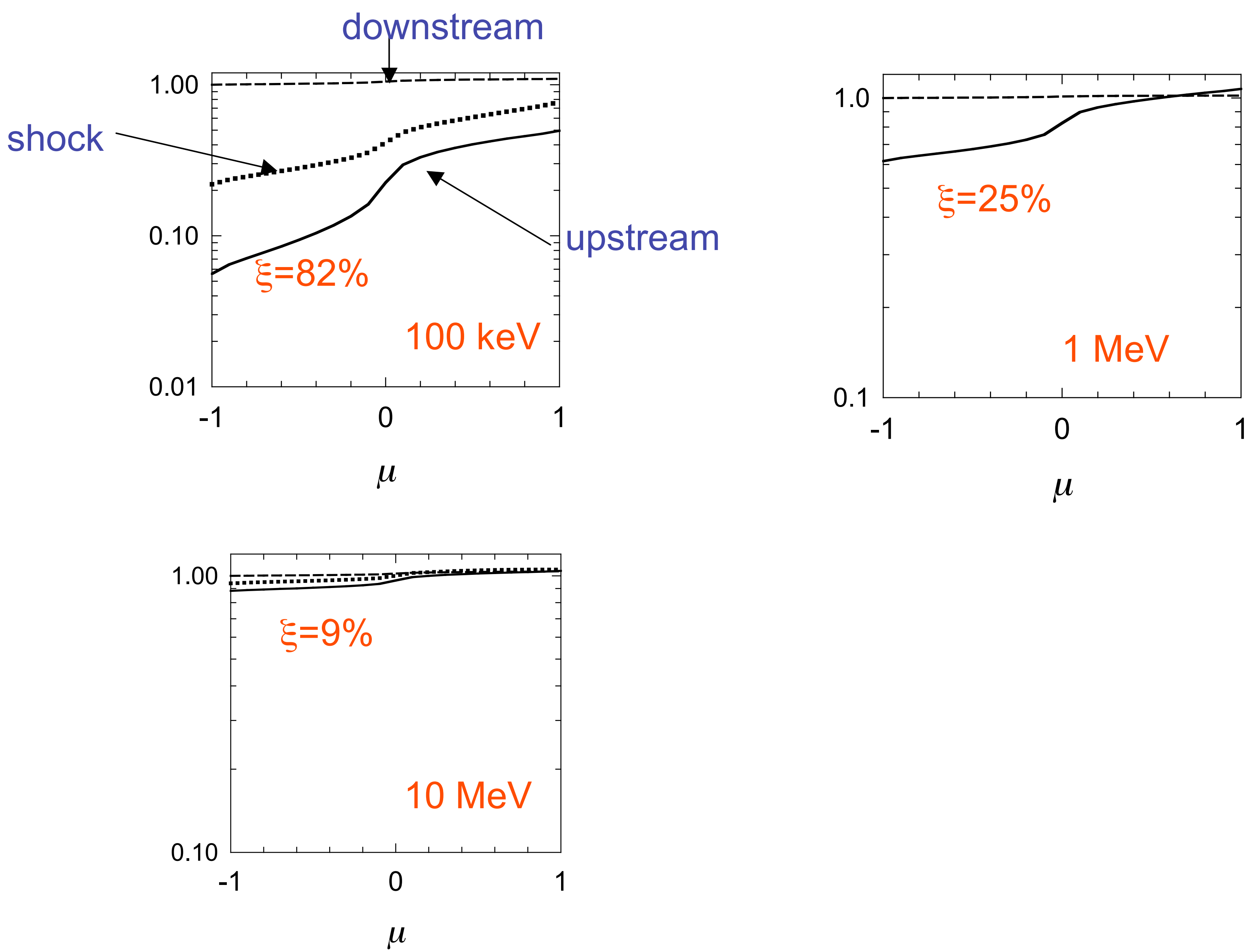
(a) Proton spectra in SW frame



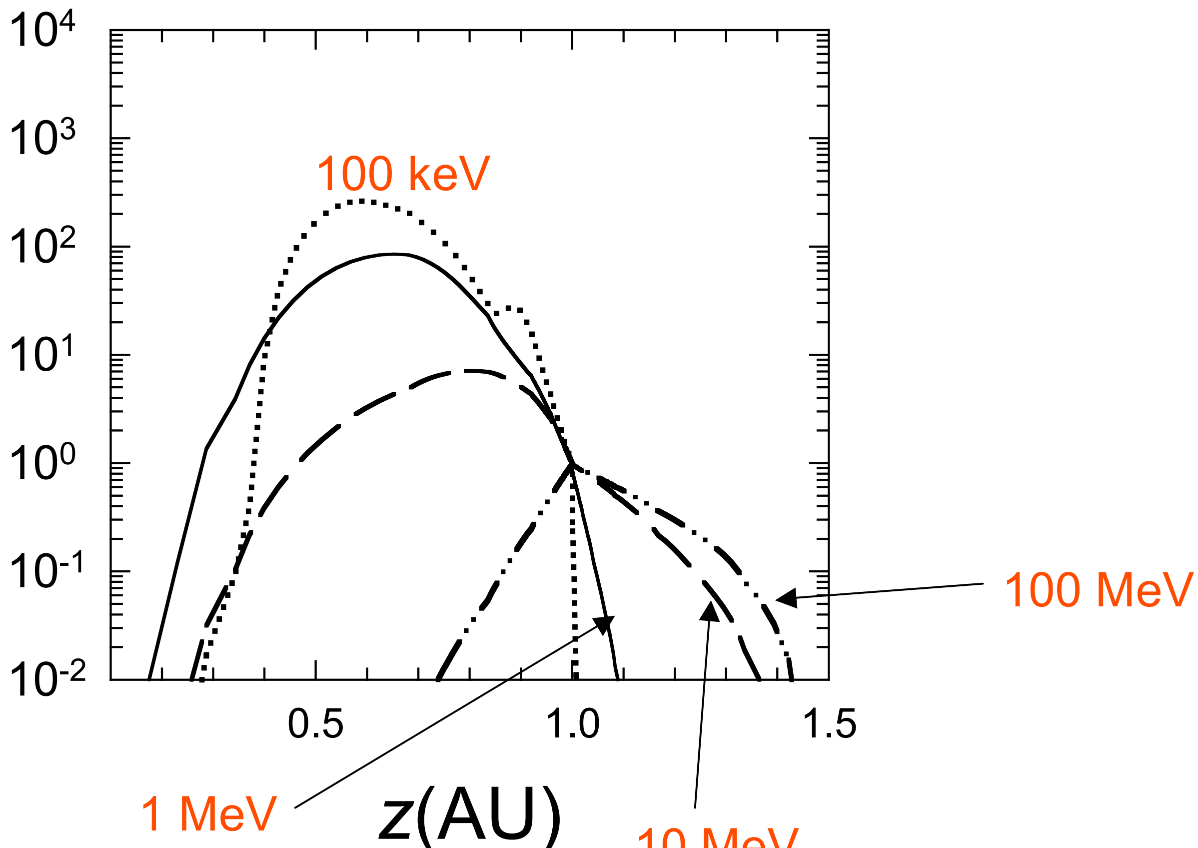


SW Kappa distribution (κ = 3) ∝ 1/r^2
Shift in peak to lower energies
$f(v) \propto v^{-5.3}$
$f(v) \propto v^{-32}$
__0.13 AU
---0.27 AU
…0.49 AU
_.._0.71 AU
____1 AU
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
v/U_e

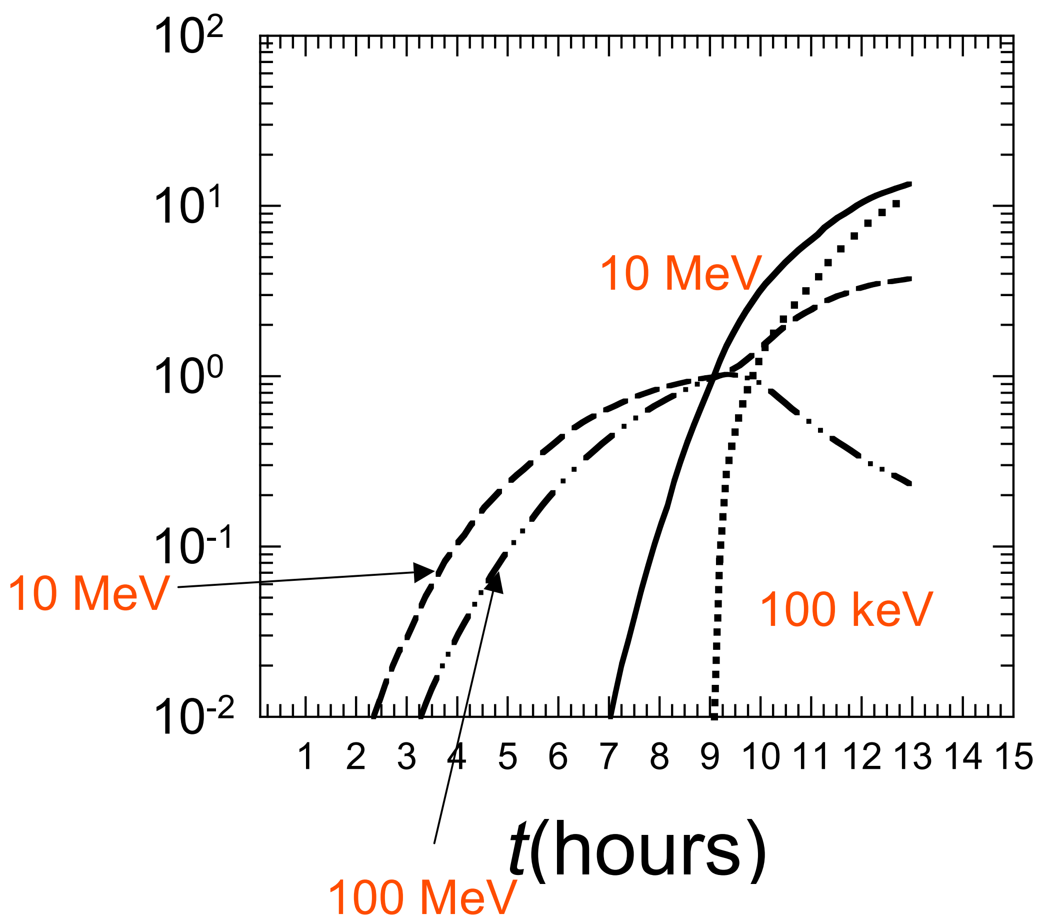
(b) Anisotropies in SW frame at 1 AU



(c) Spatial distributions across shock



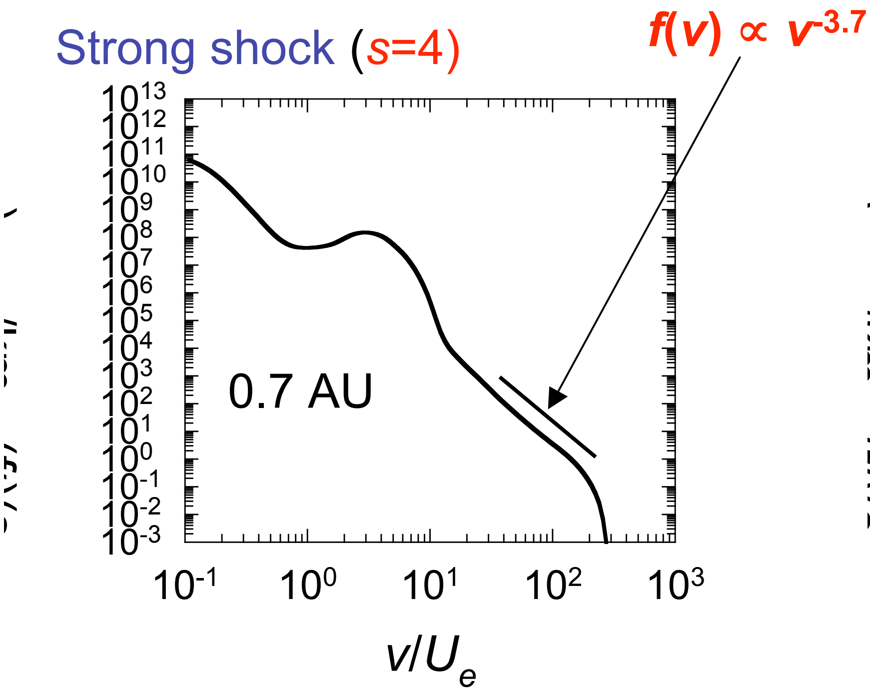
(d) Time distributions at 1 AU



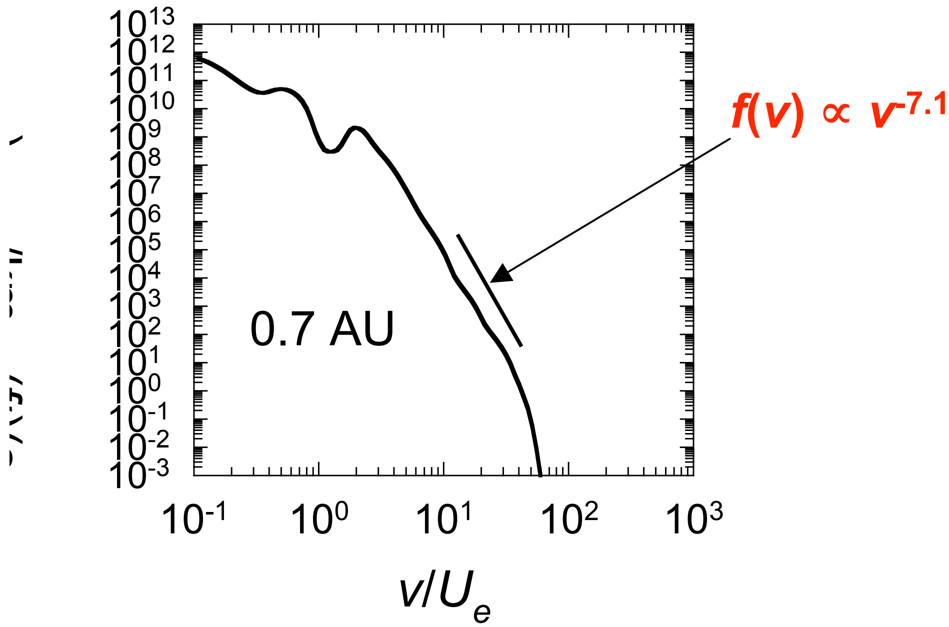
3. COMPARISON OF STRONG AND WEAK SHOCKS:

(a) Accelerated spectra

Strong shock (s=4)



Weak shock (s=3.3→2.2)

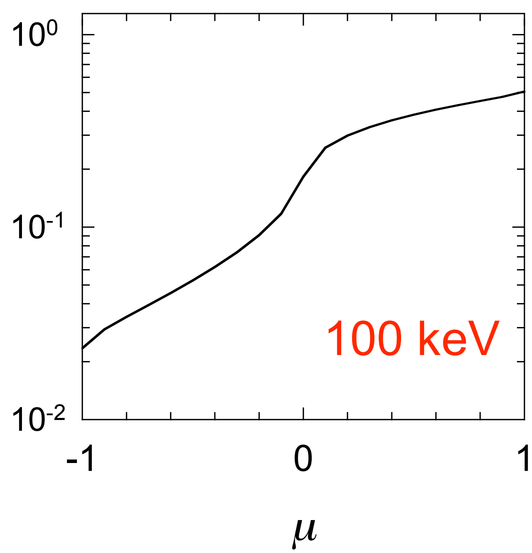


Spectrum harder than predicted by steady state DSA

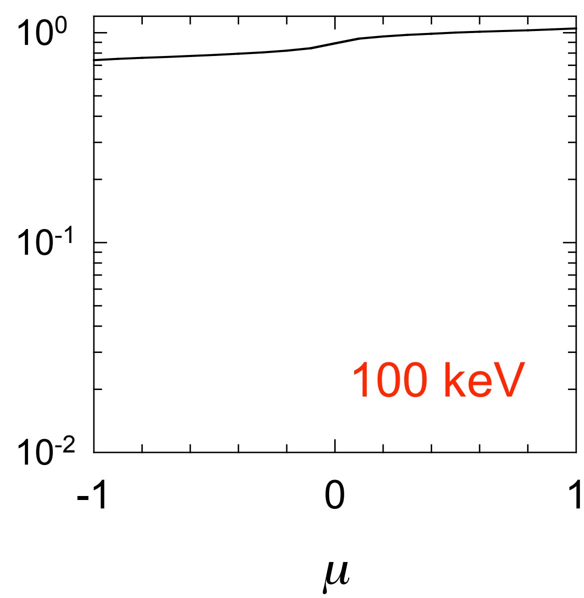
Spectrum softer than predicted by steady state DSA

(b) Anisotropies

Strong shock (s=4)



Weak shock (s=3.3→2.2)



Pitch-angle anisotropy larger for a strong shock

1D parallel Alfven wave transport equation at parallel shock solved

$$\frac{\partial I}{\partial t}$$

$$+ \left(U_{sh} + V_A\right)\frac{\partial I}{\partial z}$$

Convection

$$- \frac{\partial}{\partial z}\left(U_{sh} + V_A\right)k \frac{\partial I}{\partial k}$$

Convection in wave number space-
Compression shortens wave length

$$= -\left(\frac{1}{2}\frac{\partial U_{sh}}{\partial z}\right)I$$

Compression raises wave amplitude

$$+ \gamma(k_i)I$$

Wave generation by SEP anisotropy

$$+ \frac{\partial}{\partial k}\left(D_{kk}(I(k),k)\frac{\partial I}{\partial k}\right)$$

Wave-wave interaction – forward cascade in wave number space in inertial range

Wave-wave interaction Model (Zhou & Matthaeus, 1990):

$$D_{kk} = \frac{k^2}{t_s}$$

- *Kolmogorov cascade* $I(k) \propto k^{-5/3}$
  $t_{NL} << t_A$ $(\delta U = \delta V_A >> V_A)$ or $(\delta B >> B_o)$

$$t_s = t_{NL} = \frac{l}{\delta U}$$

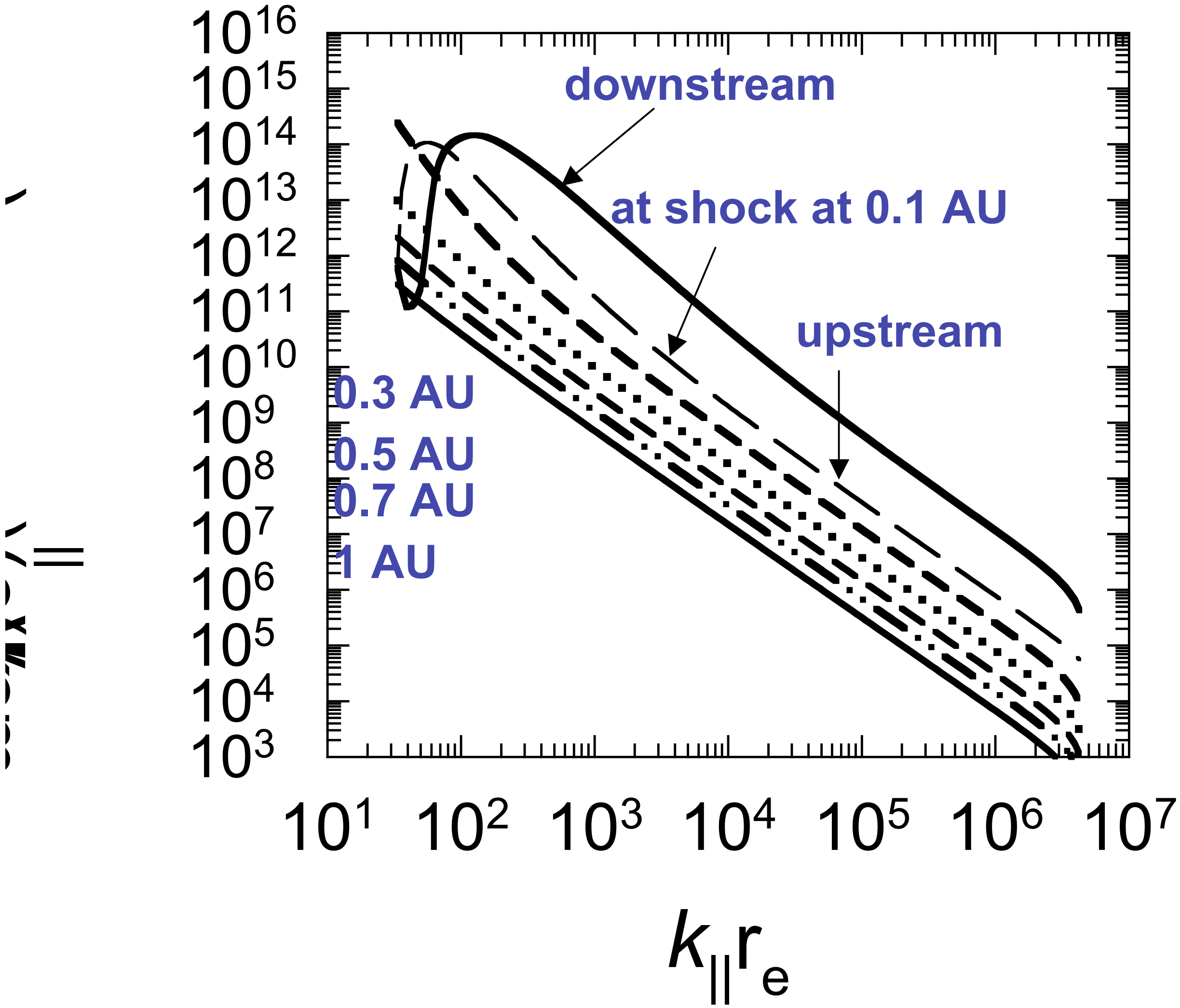
Kraichnan cascade $I(k) \propto k^{-3/2}$
$t_A << t_{NL}$ $(\delta U = \delta V_A << V_A)$ or $(\delta B << B_o)$

$$t_s = t_A = \frac{l}{V_A}$$

Comments

- Turbulence can be modeled as a nonlinear diffusion process in wavenumber space (***D_{kk}***)
- Ad-hoc turbulence model based on dimensional analysis
- Contrary to weak turbulence theory – assume that 3-wave coupling for Alfven waves is possible – imply wave resonance broadening – momentum and energy conservation in 3-wave coupling does not hold

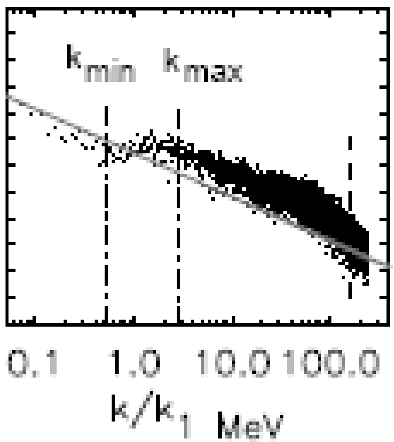
Alfven wave Spectrum (Kolmogorov Cascade)



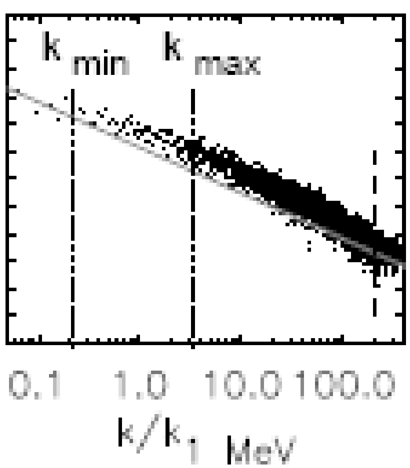
Need C_K > 8 for cascading to work

Kolmogorov or Kraichnan Cascade?

Bastille Day CME event



Halloween CME event #1



Kallenbach, Bamert, le Roux et al., 2008

Used Marty Lee (1983) quasi-linear theory extended to include nonlinear wave cascading via a diffusion term in wavenumber space.

Applied theory to Bastille Day and Halloween #1 CME events to calculate k_{min} for wave growth

Find that Kraichnan-type turbulence gives the correct k_{min}

Kolmogorov-type turbulence overestimates k_{min} by a factor 2-4

Vasquez et al. , 2007

For cascade rates and proton heating rates to agree in SW – C_K= 12 needed instead of the expected C_K ~1

Kraichnan cascade rate is close to heating rate at 1 AU

A Weak Kinetic Turbulence Theory Approach to Wave Cascading?

Wave Kinetic Equation

$$\frac{dI(k)}{dt} = 2\underbrace{\gamma(k)}_{\text{Linear wave-particle interaction}}I(k) + \overbrace{\int dk'[|a_{k,k'}|^2 \underbrace{I(k')I(k'')}_{\text{Wave generation}} - 2\underbrace{a_{k,k'}a_{k',k}}_{\text{Wave decay}} I(k)I(k'')]\delta(\omega(k)-\omega(k')-\omega(k''))}^{\text{Nonlinear wave-wave interaction}}$$

Wave-wave interaction term a Boltzmann scattering term

When assume weak scattering, term can be put in Fokker-Planck form so that nonlinear diffusion coefficients for wave cascading D_{kk} can be derived