Exponential Rollovers

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Shock Acceleration I

$$V_z \frac{\partial f_i}{\partial z} - \frac{\partial}{\partial z}\left(K_{i,zz}\frac{\partial f_i}{\partial z}\right) - \frac{1}{3}\frac{dV_z}{dz} v \frac{\partial f_i}{\partial v} = Q_i$$

$$\zeta(z) = \int_0^z [V/K_{zz}(z')]dz'$$

$$f(z>0,v) = f_0 - (f_0 - f_\infty)(1-e^{-\zeta})(1-e^{-\zeta_\infty})^{-1}$$

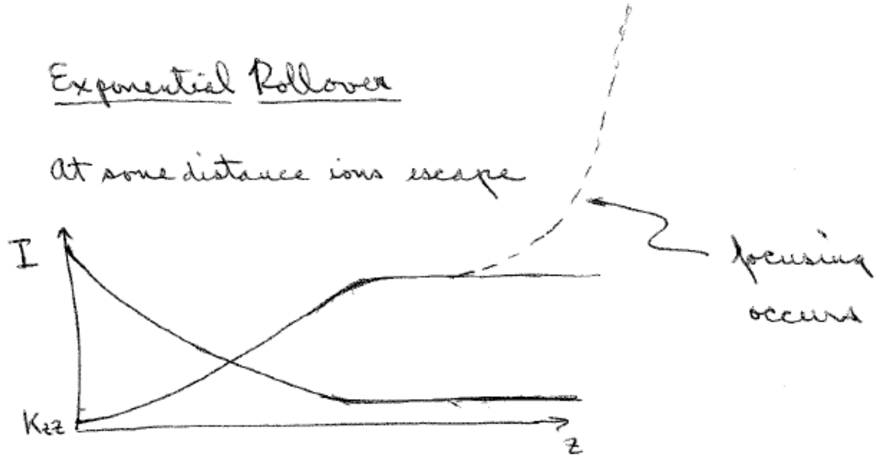
$$f_0 \equiv \beta \int_0^v \frac{dv'}{v'} f_\infty(v')(1-e^{-\zeta'_\infty})^{-1}\left(\frac{v}{v'}\right)^{-\beta} \exp\left[-\beta \int_{v'}^{v} \frac{dv''}{v''} e^{-\zeta''_\infty}(1-e^{-\zeta''_\infty})^{-1}\right]$$

Shock Acceleration II

$$-K_{zz}\frac{\partial f}{\partial z}\bigg|_{z\to\infty} = V(f_0 - f_\infty)e^{-\zeta_\infty}(1-e^{-\zeta_\infty})^{-1}$$

Exponential Rollover

at some distance ions escape



focusing when

$$\frac{(1-\mu^2)}{\Omega}(v+\mu U)\frac{\partial f}{\partial \mu} \approx \frac{\partial}{\partial \mu}\left[(1-\mu^2)D_{\mu\mu}\frac{\partial f}{\partial \mu}\right]$$

$$\frac{v}{\Omega} \simeq D_{\mu\mu} = \frac{\pi}{2}\frac{\Omega^2}{B_0^2}\frac{1}{v} I\left(\frac{\Omega}{v}\right)$$

at this point there is an effective free-escape boundary

upstream Parker equation:

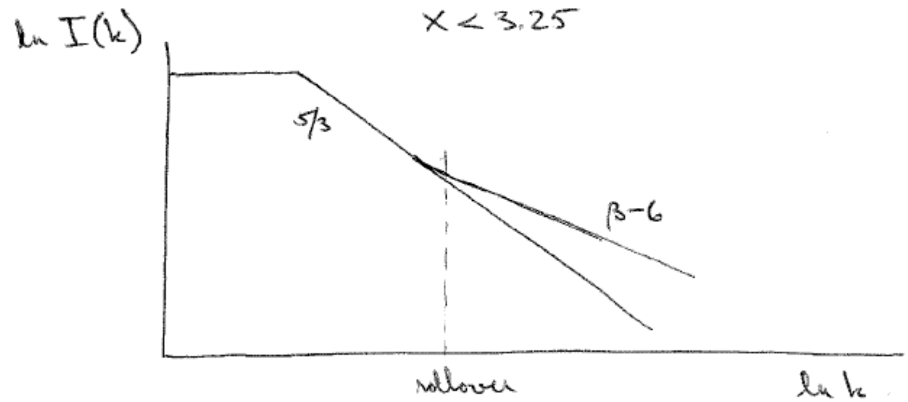
$$-V\frac{\partial f}{\partial z} - \frac{\partial}{\partial z}\left(K_{zz}\frac{\partial f}{\partial z}\right) = 0$$

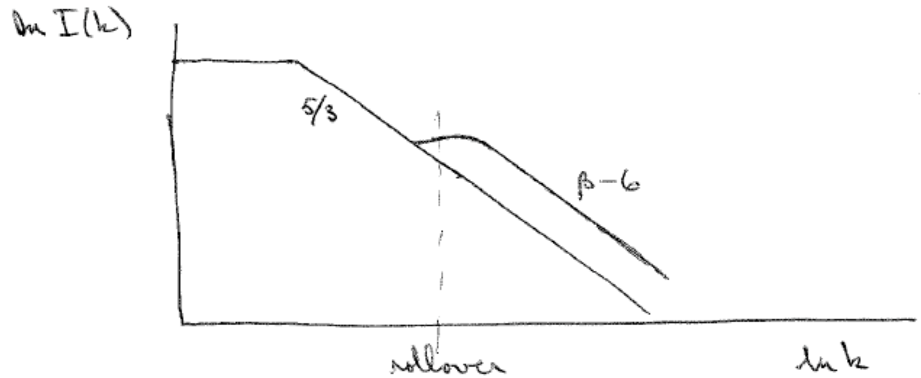
$$-\frac{K_{zz}}{V}\frac{\partial f}{\partial z} - \frac{K_{zz}}{V}\frac{\partial}{\partial z}\left(\frac{K_{zz}}{V}\frac{\partial f}{\partial z}\right) = 0$$

$$\zeta = \int_0^z \frac{V\,dz}{K_{zz}}$$

$$\zeta_{es}\ (\text{or } \zeta_\infty) = \int_0^{z_{es}} \frac{V\,dz}{K_{zz}}$$

not sensitive since K_{zz} large for $I = I_0$





What is $I(\Omega/v)$ at rollover averaged over z

$$S_\infty = \int_0^{z_{osc.}} V\,dz\ \frac{8\pi\Omega^2}{v^3}\frac{1}{\cos^2\psi}\, I\!\left(\frac{\Omega}{v}, z\right)$$

If $I \propto k^{-\lambda}$

$$S_\infty \propto \left(\frac{v}{\Omega}\right)^{\lambda-2}\frac{1}{v}$$

$$\exp\left[-\beta\int_{v_0}^{v}\frac{dv'}{v'}\,e^{-\zeta_\infty'}\,\frac{1}{1-e^{-\zeta_\infty'}}\right]$$

$$\zeta_\infty = \int_0^{z_{esc}}\frac{V\,dz'}{K_{zz}(z',v)}$$

Integrand only important for $\zeta_\infty \lesssim 1$

$$\Rightarrow \quad \exp\left[-\beta\int_{v_0}^{v}\frac{dv'}{v'}\,\frac{1}{\zeta_\infty'}\right]$$

$$\sim \exp\left[-\beta\,\frac{1}{z_{esc}}\int_{v_0}^{v}\frac{\langle K_{zz}\rangle}{V}\,\frac{dv'}{v'}\right]$$

Time limitation of DSA

$$\sim \exp\left[-\beta\,\frac{1}{Vt}\int_{v_0}^{v}\frac{dv'}{v'}\,\frac{K_{zz}}{V}\right] \qquad \text{constant } K_{zz}$$

As long as $z_{esc} < Vt$ escape should dominate

note $Vt < V_{sw}t \sim r_s$

When does focusing dominate?

Ambient waves?

$$\frac{1}{\lambda} : \frac{\Omega^2}{B_0^2} \frac{1}{v^2} I_0 \left(\frac{\Omega}{v k_0}\right)^{-5/3}$$

At very high energies $\Omega/v = k_0$

$$\frac{1}{\lambda} : \frac{k_0^2}{B_0^2} I_0 = \frac{1}{2} k_0 \left(\frac{2 k_0 I_0}{B_0^2}\right) = \frac{\pi}{\lambda} \frac{\langle|\delta B|^2\rangle}{B_0^2} \simeq \frac{1}{\lambda}$$

But only 10% of fluctuations are Alfvenic!

Implies equality of mirror and scattering

Velocity dependence of scattering

$$v^{-1/3} \propto E^{-1/6} \quad \text{very weak!}$$

Exponential Rollover I

$$\exp\left[-\beta\int_0^v \frac{dv'}{v'}\frac{e^{-\zeta'_\infty}}{1-e^{-\zeta'_\infty}}\right] \qquad \zeta_\infty = \int_0^\infty dz\,\frac{V}{K_{zz}}e^{-z/z_s}$$

$$\zeta_\infty = V\int_0^\infty dz e^{-z/z_s}\left[\cos^2\psi\,\frac{v^3}{4\pi}\frac{B_0^2}{\Omega_i^2}\int_{-1}^1 d\mu\,\frac{|\mu|(1-\mu^2)}{I(\Omega_i/v\mu)}+\sin^2\psi K_\perp\right]^{-1}$$

If $I(k)\propto k^{-\lambda}, K_\perp \approx 0$

Exponential Rollover II

$$\exp\left[-Cv^{3-\lambda}(Q/A)^{\lambda-2}\right]$$

Not exponential
In energy!

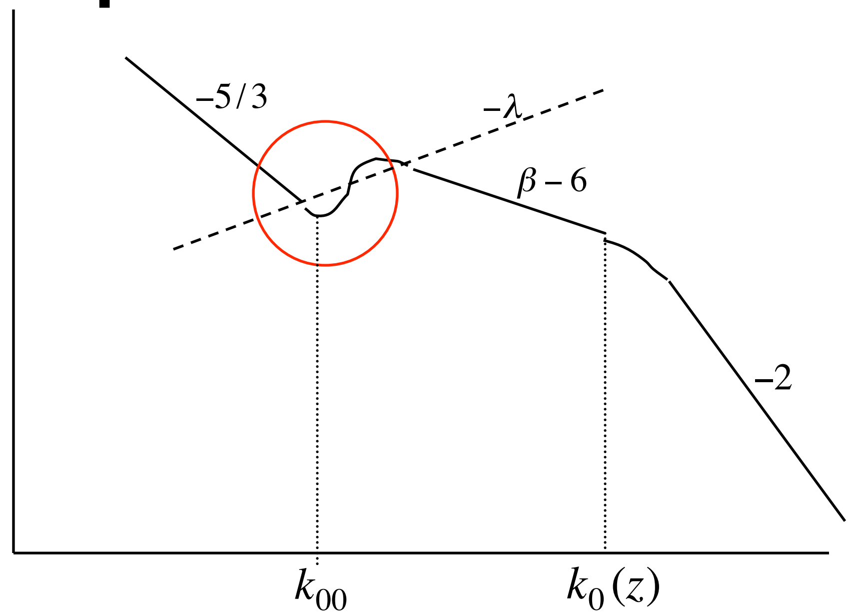
$$E_0 \propto (Q/A)^b \qquad b = 2\,\frac{2-\lambda}{3-\lambda}$$

$$b(5/3) = 1/2$$

$$b(\lambda \to -\infty) = 2$$

Exponential Rollover III

I(k)?

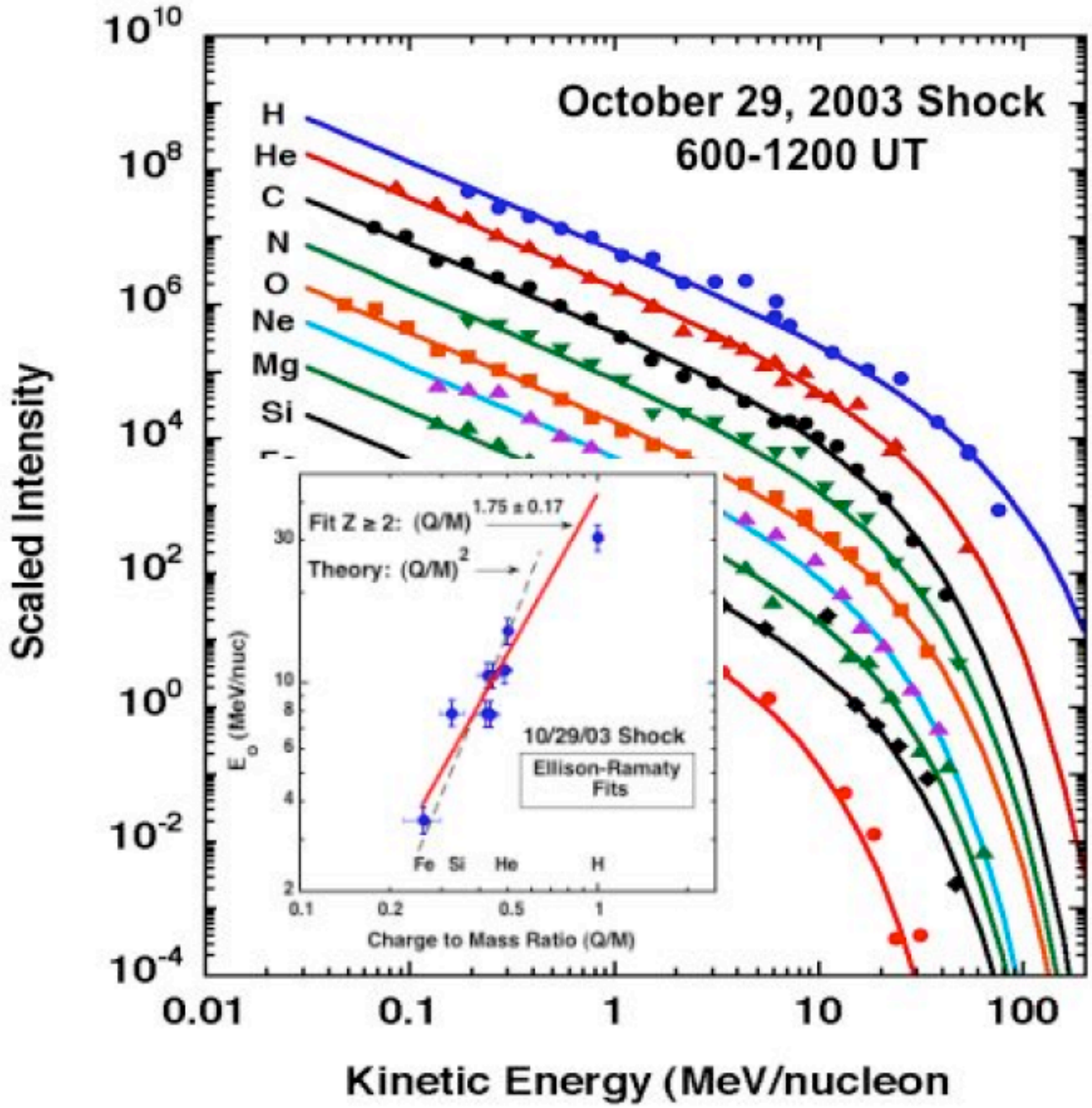


ζ_∞ Involves integration over μ, z

Weak events: $\beta/2 - 1 \geq 2 \Rightarrow \lambda \approx 5/3 \Rightarrow b \approx 1/2$ (*Cohen et al., 2005*)

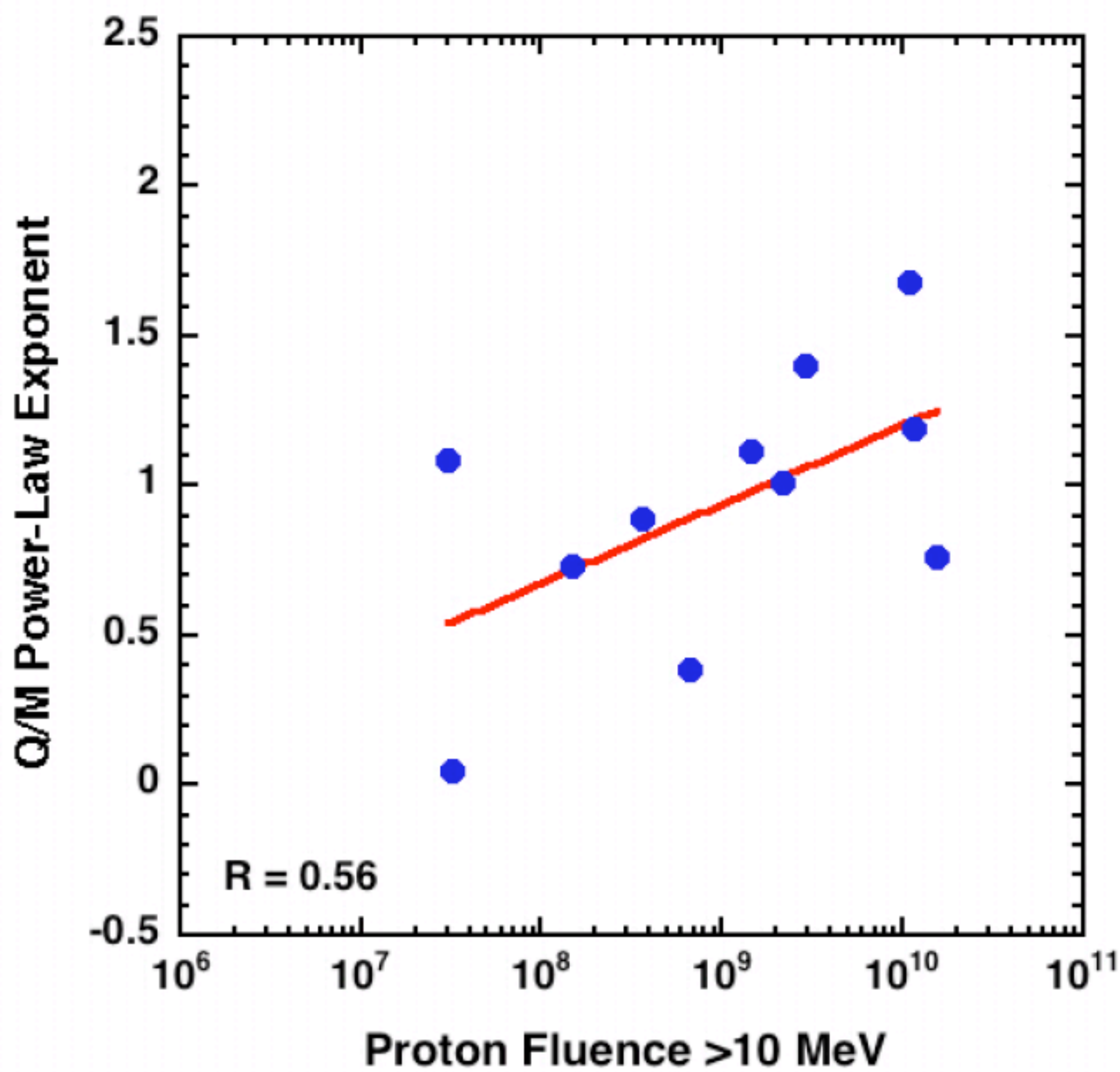
Strong events: $\beta/2 - 1 \approx 1 \Rightarrow \lambda \to \infty \Rightarrow b \approx 2$ (*Li et al., 2007*)

SEP Spectral Rollover Fits I



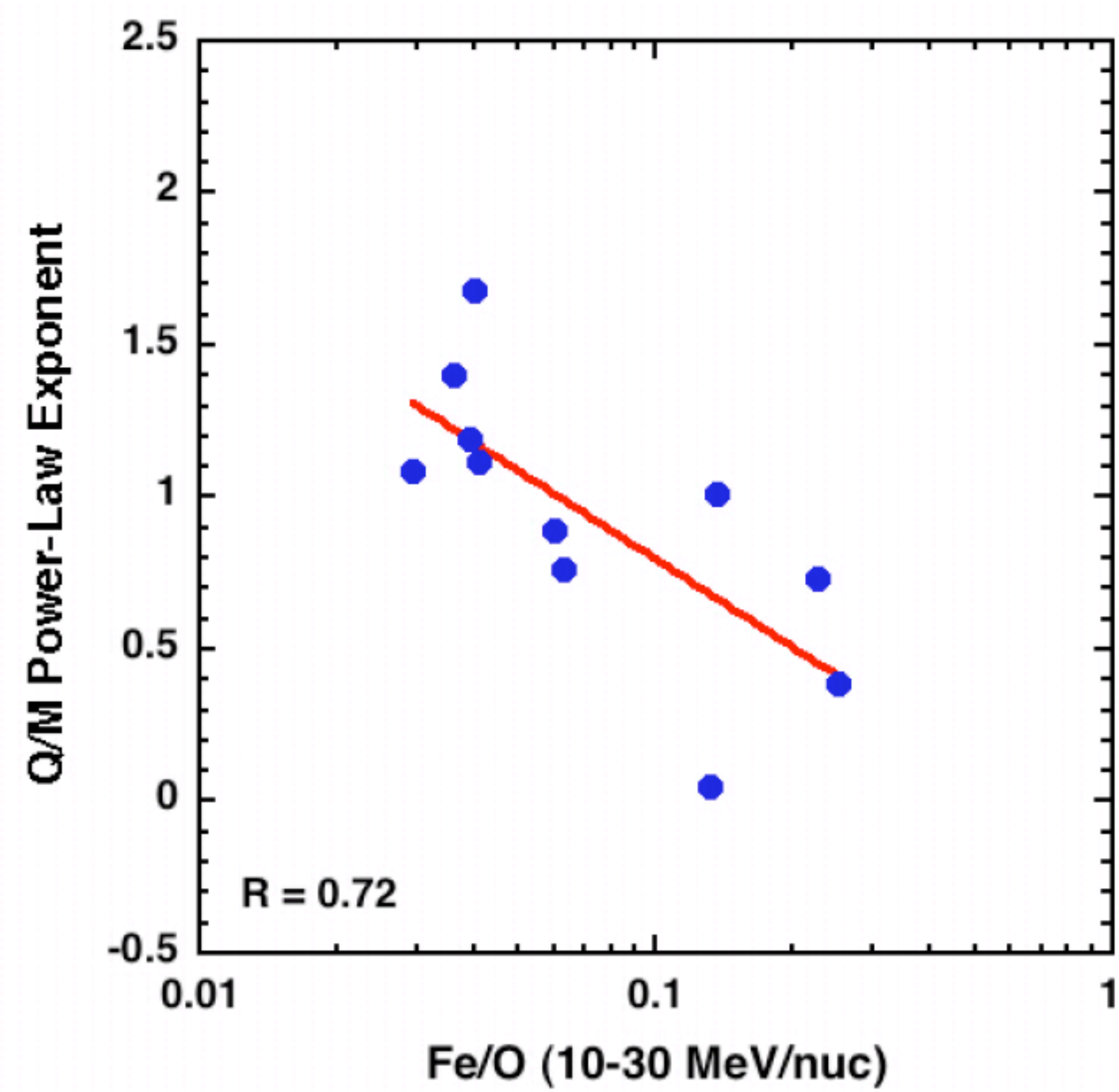
Mewaldt et al., 2007

SEP Spectral Rollover Fits II



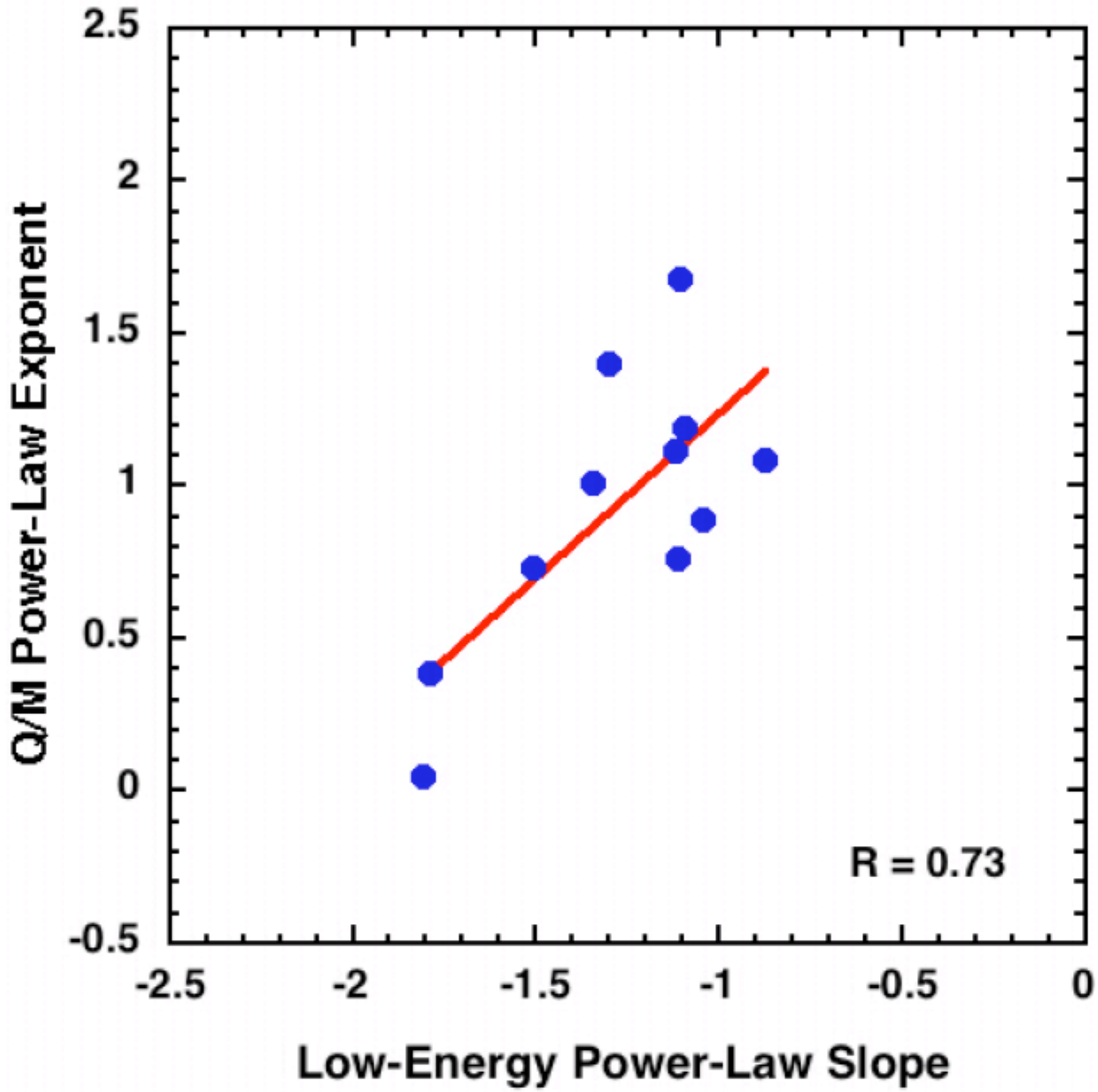
Mewaldt et al., 2007

SEP Spectral Rollover Fits III



Mewaldt et al., 2007

SEP Spectral Rollover Fits IV



Mewaldt et al., 2007

Exponential Rollover III

$f_\infty(v) \propto v^{-\gamma}$ (assume wave spectrum at shock)

$\gamma > \beta$:

$$\exp\left\{-\int_0^v \frac{dw}{w}\left[\int_0^\infty dz e^{-z/z_s}\left((\sin\psi)^2 K_\perp + \int_{-1}^1 d\mu \frac{|\mu|(1-\mu^2)}{I_e w^{3-\beta}(Q/A)^{\beta-4}(\cos\psi)^{\gamma-\beta-1}\mu^{6-\beta} + I_a w^{-4/3}(Q/A)^{1/3}(\cos\psi)^{-2}\mu^{5/3}}\right)^{-1}\right]^{-1}\right\}$$

$\beta > \gamma$:

$$\exp\left\{-\int_0^v \frac{dw}{w}\left[\int_0^\infty dz e^{-z/z_s}\left((\sin\psi)^2 K_\perp + \int_{-1}^1 d\mu \frac{|\mu|(1-\mu^2)}{I_e w^{3-\gamma}(Q/A)^{\gamma-4}(\cos\psi)^{-1}\mu^{6-\gamma} + I_a w^{-4/3}(Q/A)^{1/3}(\cos\psi)^{-2}\mu^{5/3}}\right)^{-1}\right]^{-1}\right\}$$