The Injection Speed - Standard Focused Transport Theory

In focused transport theory – particles follow magnetic field lines- minimum requirement for shock acceleration to work – the particles must be able to swim upstream in order to have multiple shock encounters

From the spherically-symmetric focused transport equation using the method of characteristics one gets for the radial component of the particle speed in the traveling shock frame:

$$v_r^{sh} = U_1^{sh} + v'_1 \mu'_1 \cos\psi_1 > 0$$

where U_1^{sh} is the upstream solar wind speed in the shock frame, v'_1 is the particle speed in the upstream solar wind frame and ψ_1 is the magnetic field spiral angle upstream and μ'_1 is the cosine of the particle pitch angle.

$$\Rightarrow v_1' \mu_1' > -\frac{U_1^{sh}}{\cos\psi_1}$$

However, $U_1^{sh} = U_1 - V_{sh}$, and $\psi_1 = \theta_{BN}$ for a spherical shock, where V_{sh} is the shock speed, U_1 is the upstream solar wind speed in the observer frame, and θ_{BN} is the shock normal angle.

The minimum speed for injection is

$$\Rightarrow \mathbf{v'_{min}} = \mathbf{v_{inj}} > \frac{\mathbf{V_{sh} - U_1}}{\mathbf{\cos\theta_{BN}}}$$

which is the de Hoffmann-Teller speed in the upstream solar wind frame for a traveling shock

Theoretical prediction for injection speed for strong shock moving at 2400 km/s at 0.1 AU

$$v_{inj} = \frac{V_{sh} - U_1}{\cos\theta_{BN}}$$

$$= \frac{2400km/s - 400km/s}{\cos 0^0}$$

$$= 2000km/s$$

$$\Rightarrow v_{inj}/U_e = 2000km/s/400km/s = 5$$

***Vinj/Ue* ~ 5 (v/400 km/s)** **at 0.1 AU – model solution – agrees with theoretical Prediction**