An Analytical Theory of Diffusive Shock Acceleration for Gradual SEP Events

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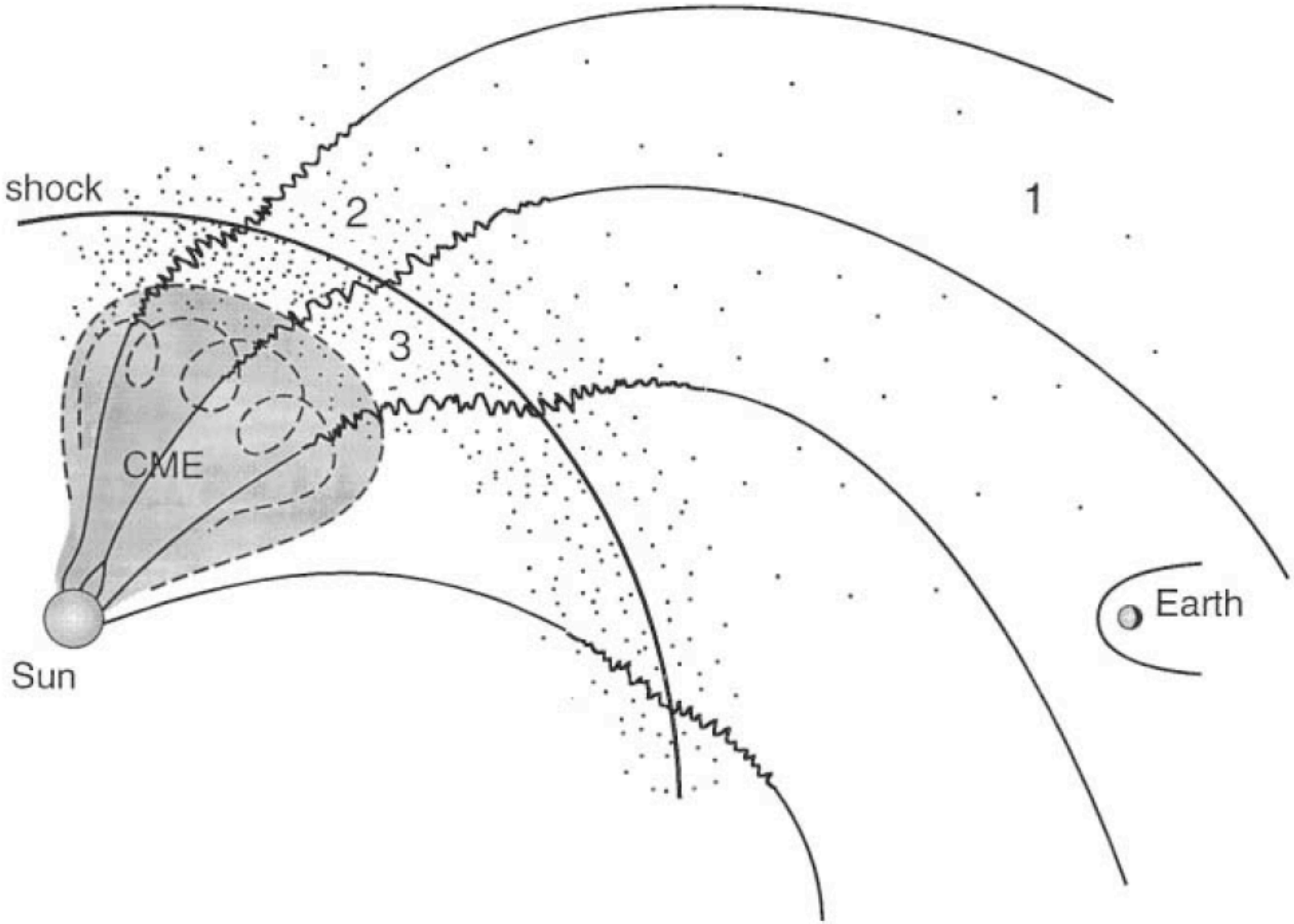
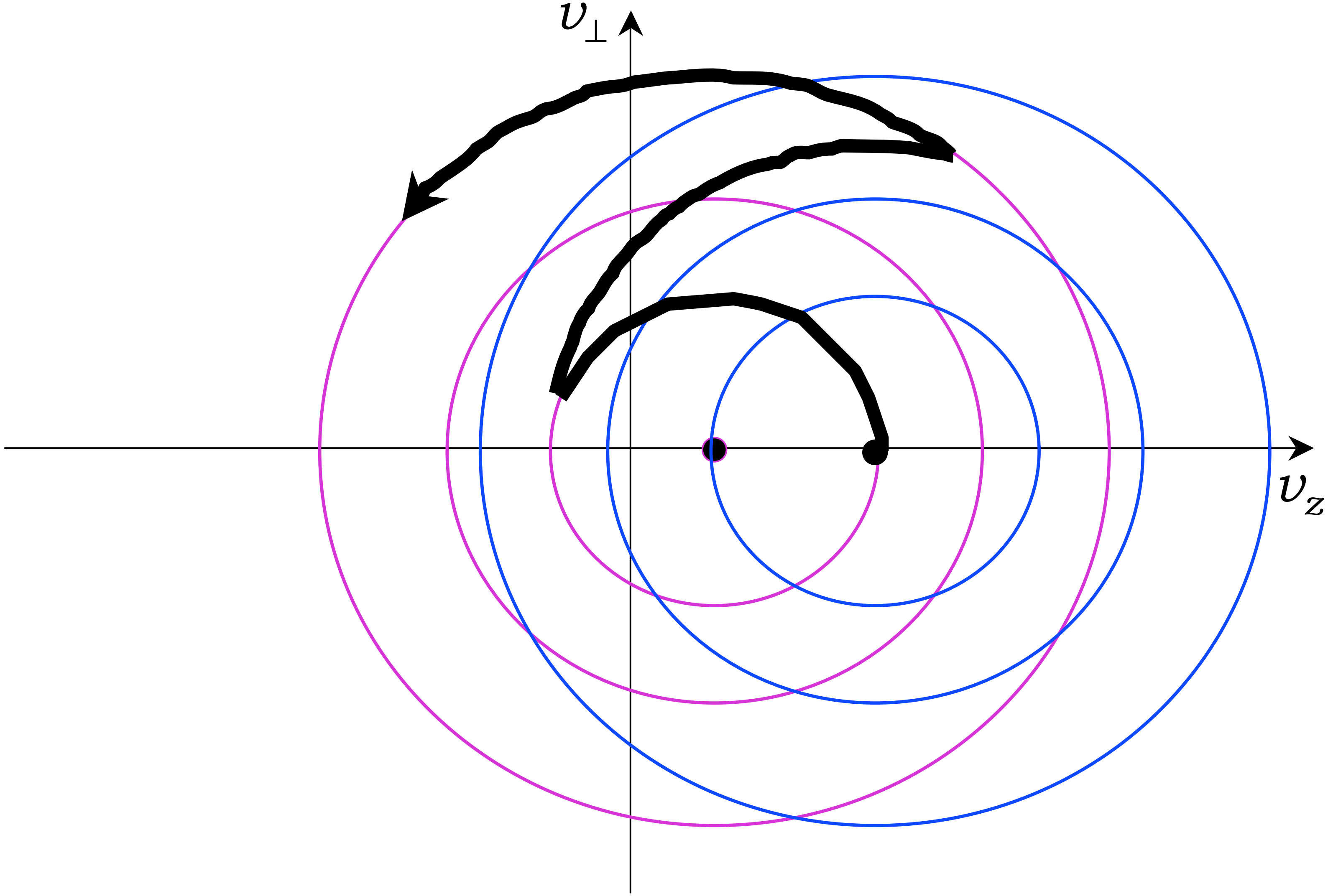
Acceleration at a CME-Driven Shock

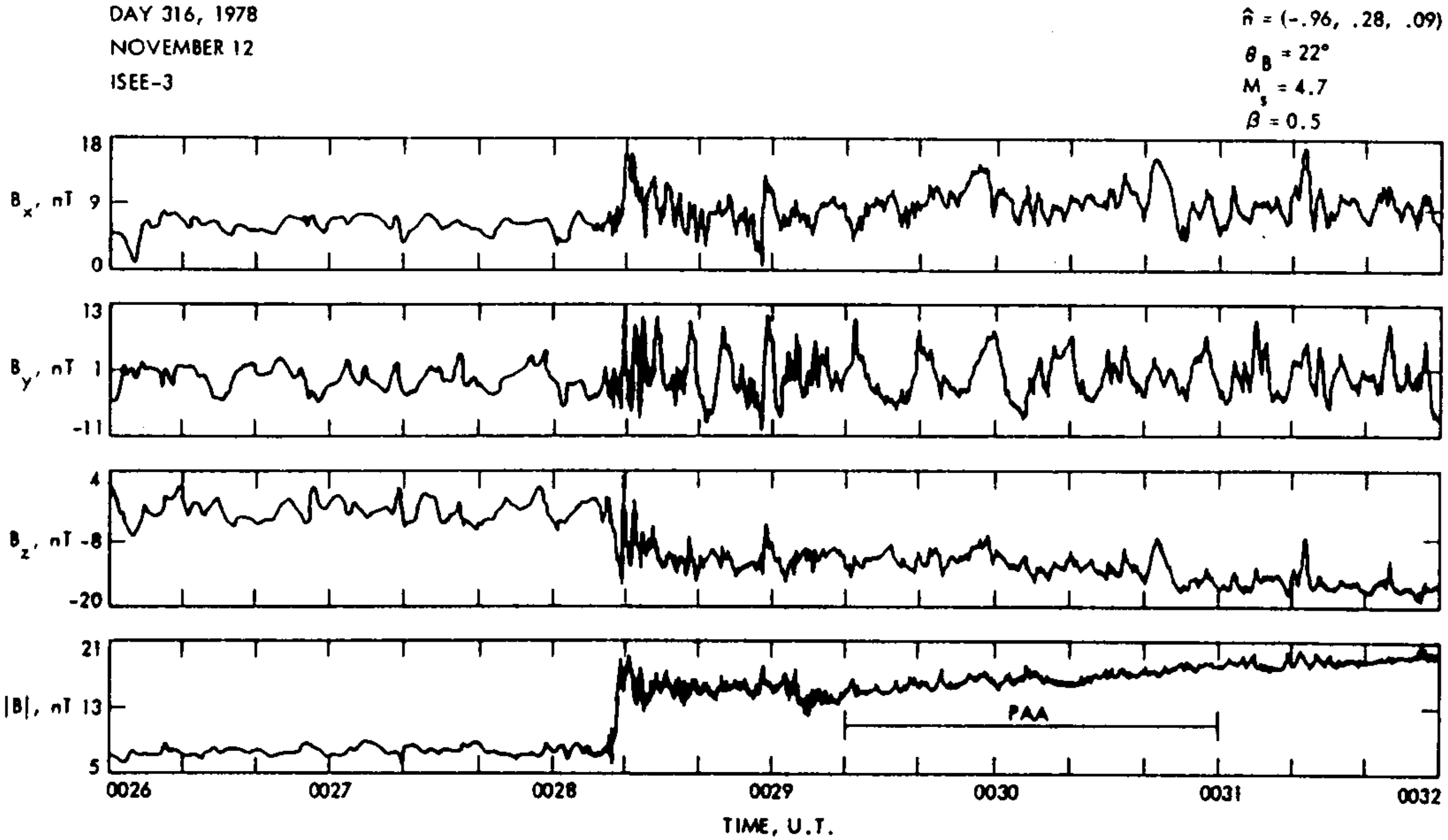


Lee, 2005

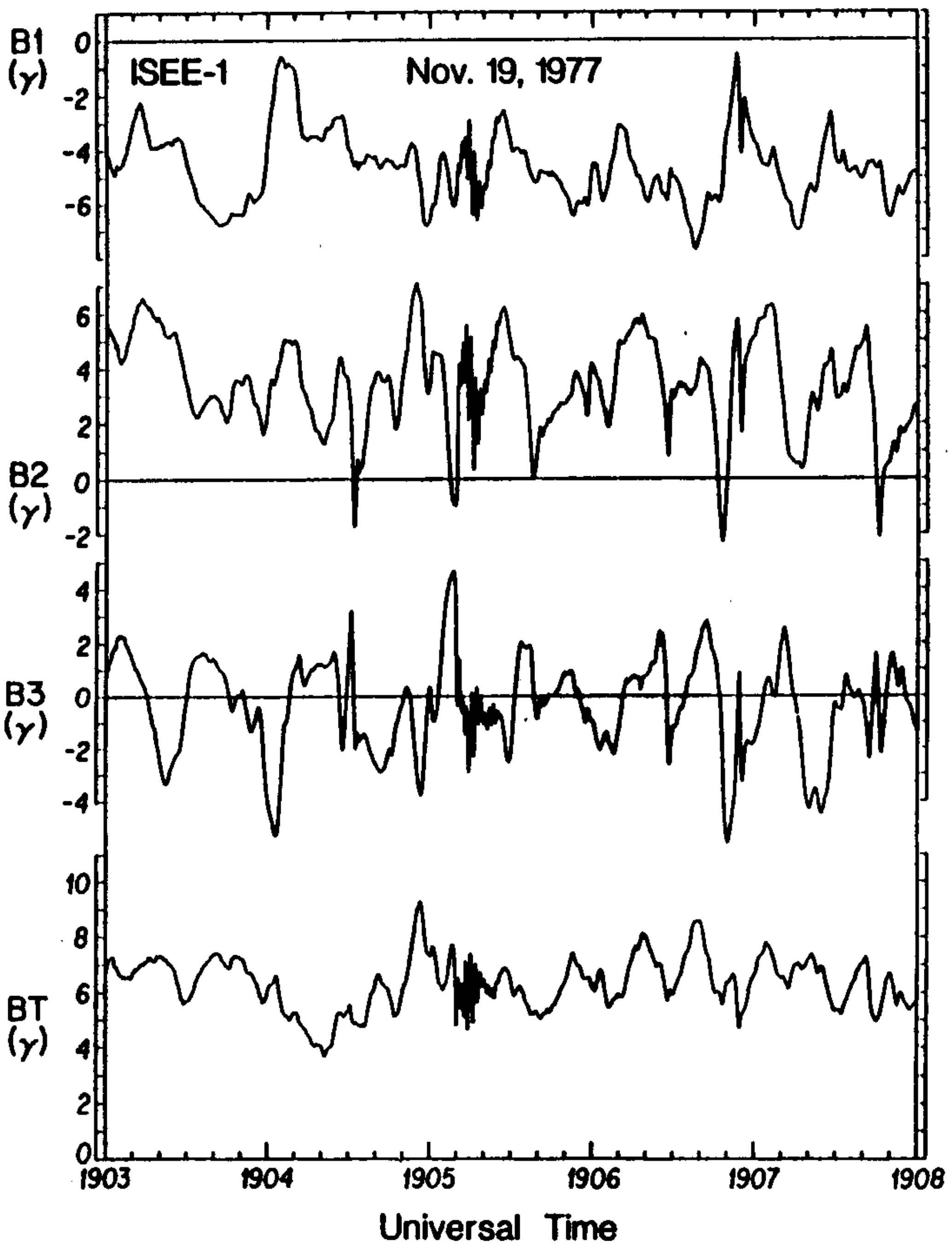
First-Order Fermi Acceleration



Upstream Waves I



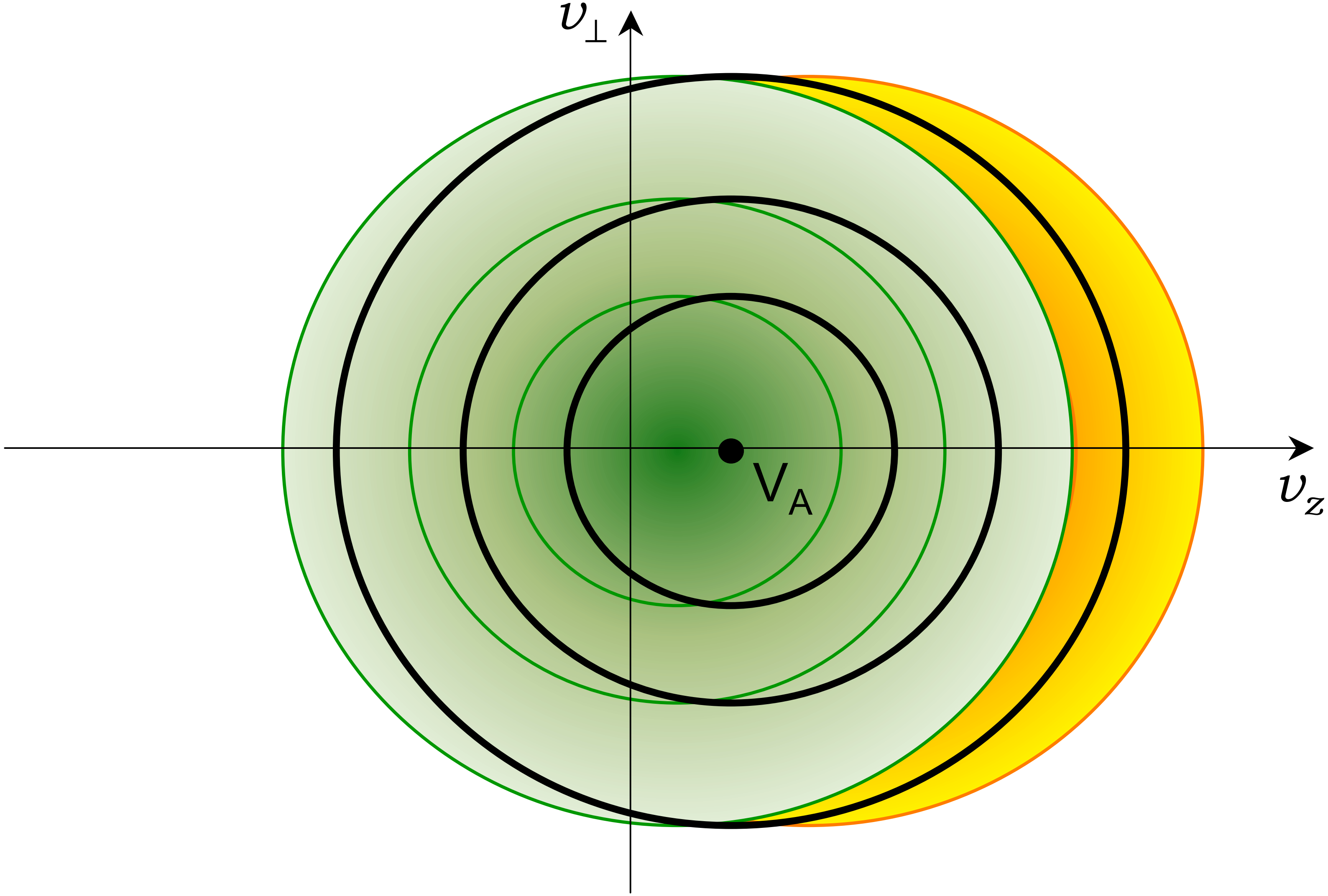
Tsurutani et al., 1983



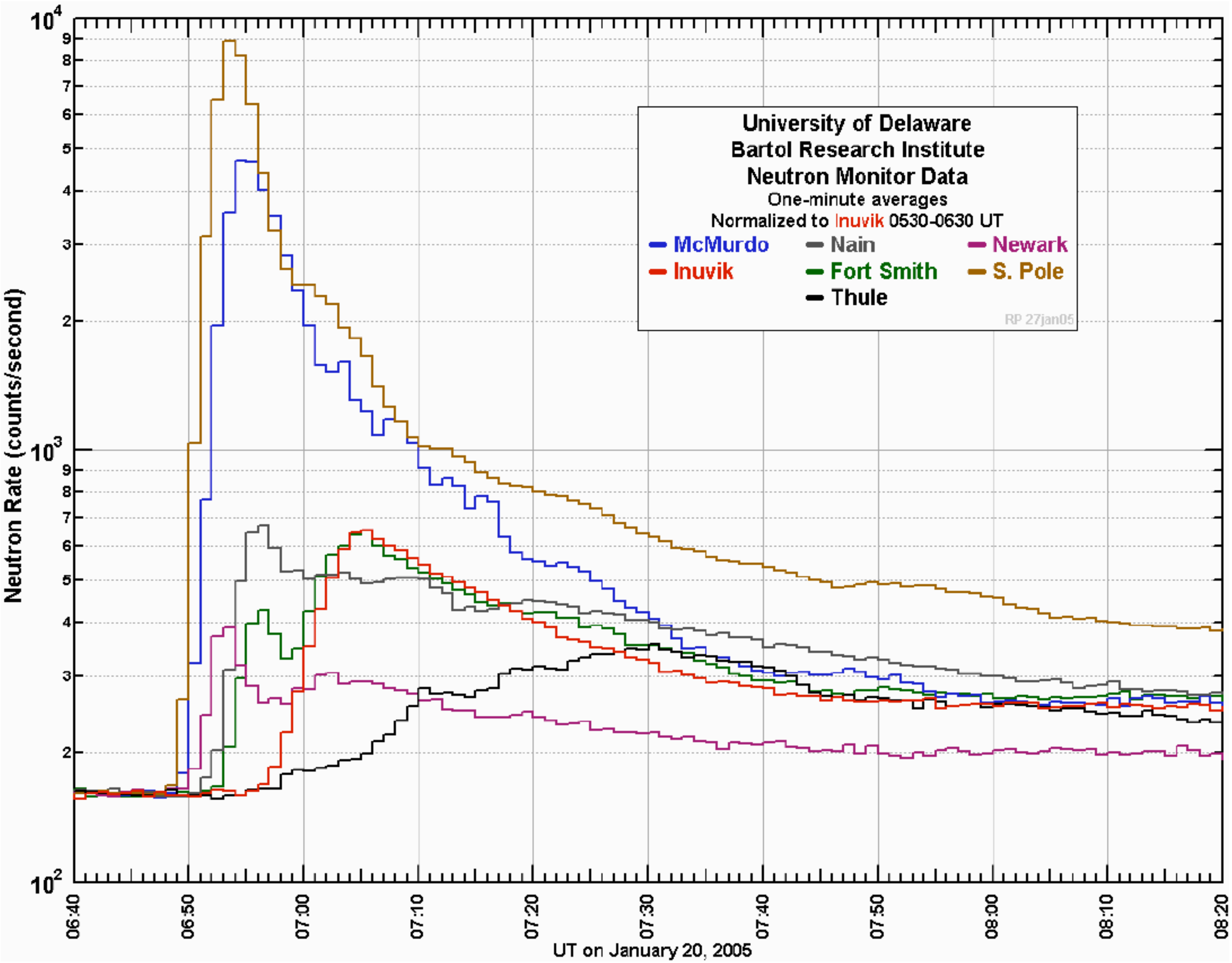
Upstream Waves II

Hoppe et al., 1981

Instability Mechanism

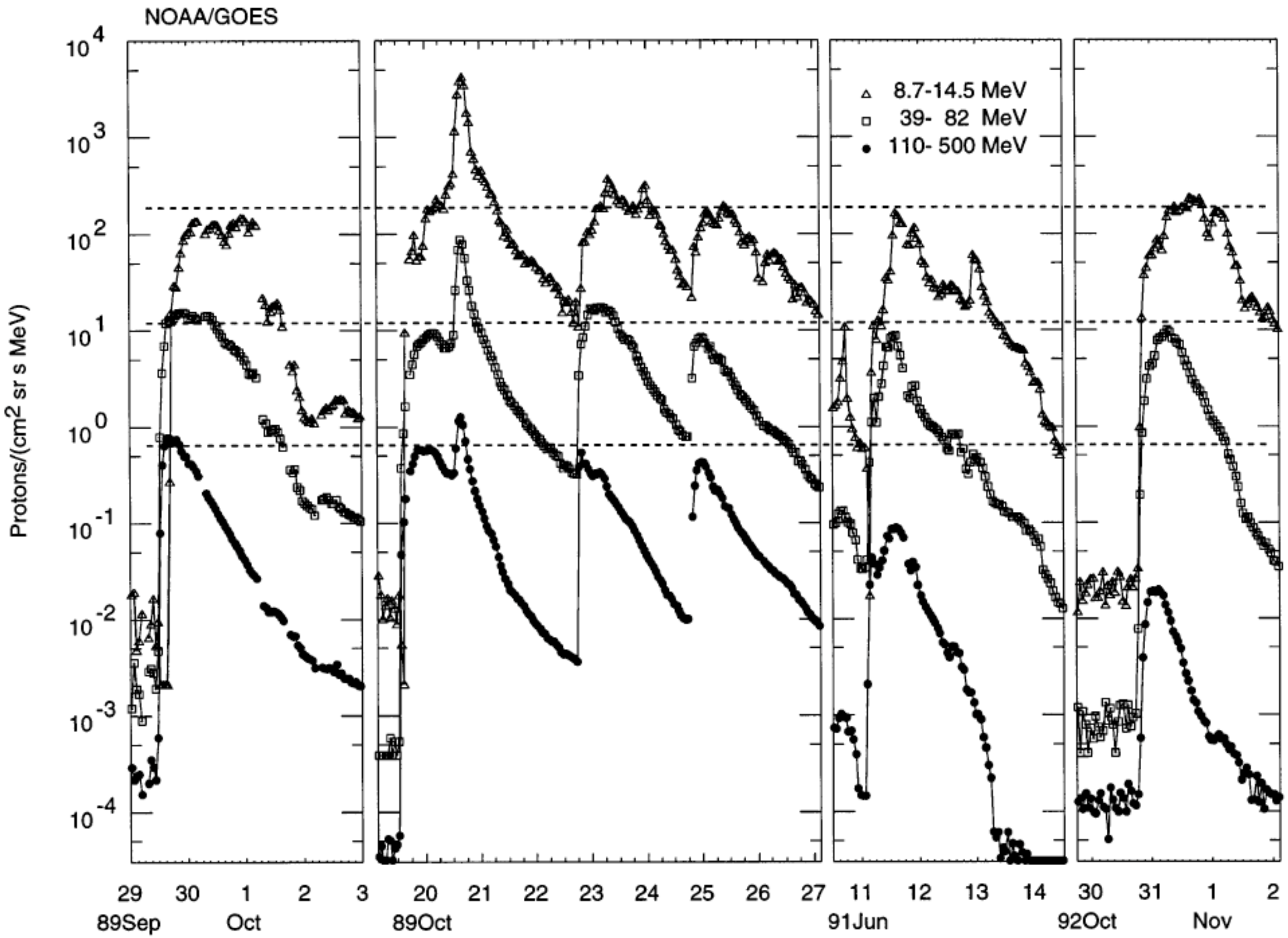


January 20, 2005 GLE Event

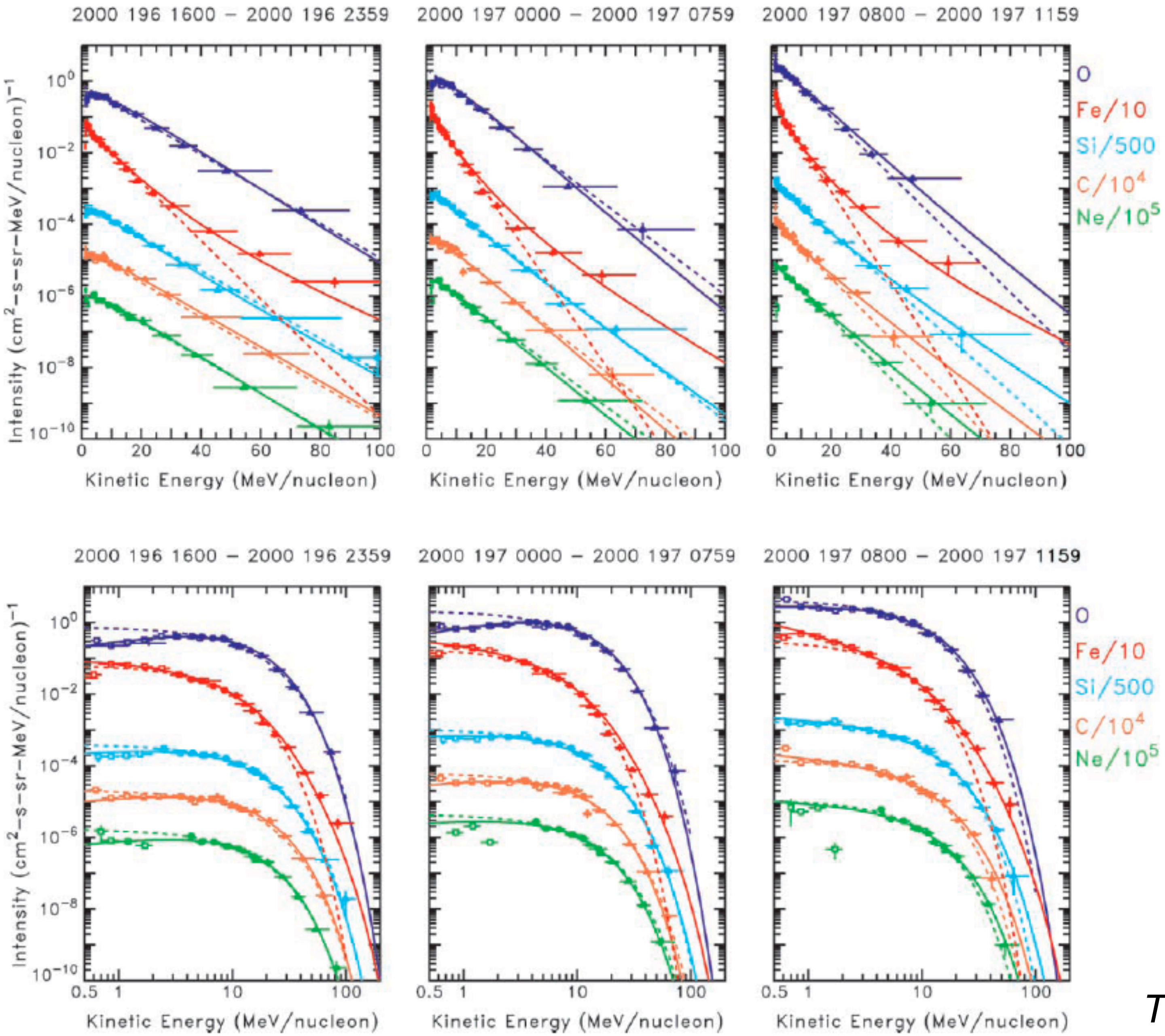


Bieber et al., 2005

Large SEP Events



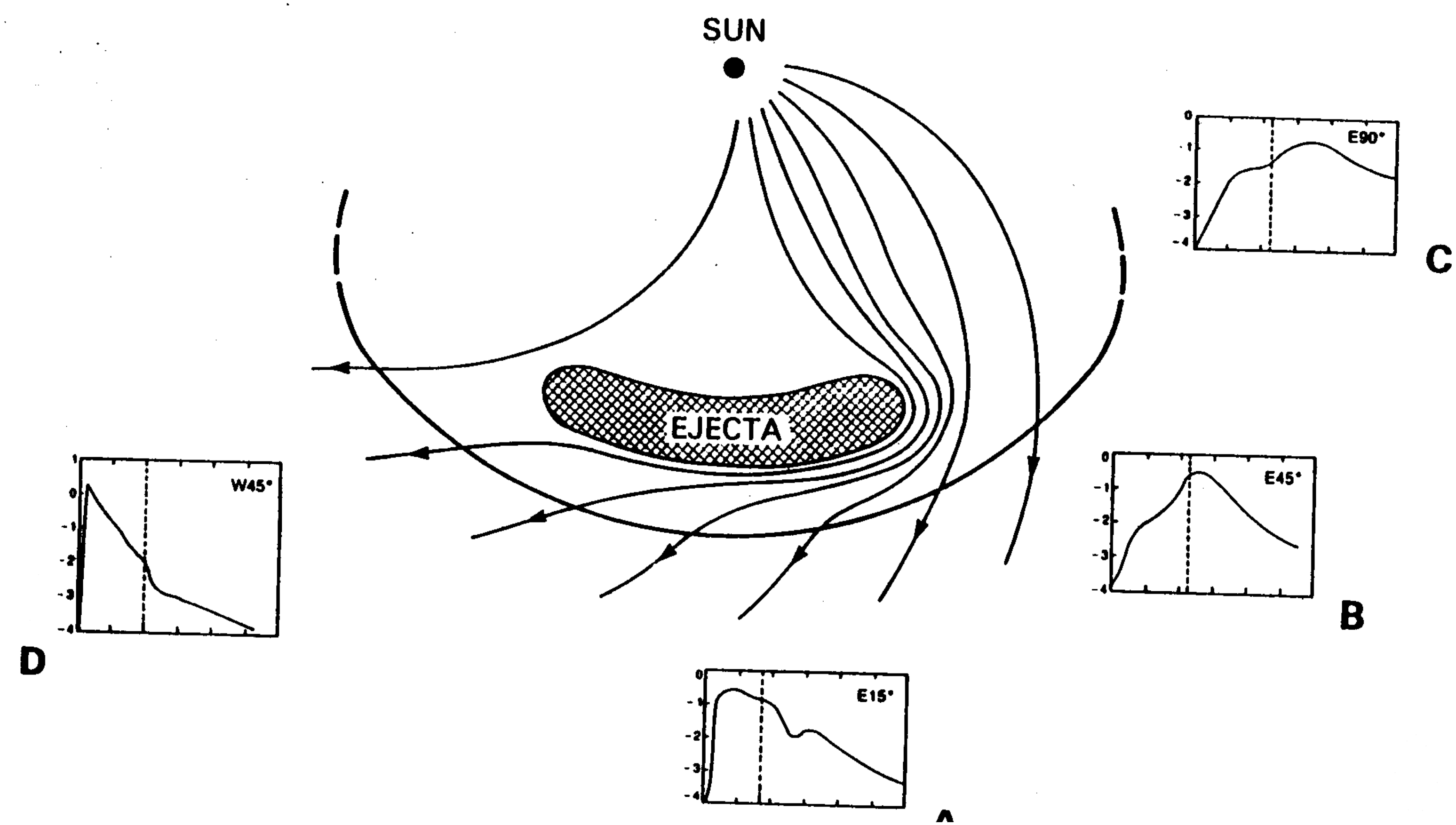
Reames and Ng, 1998



SEP Energy Spectra

Tylka et al., 2001

Shock Geometry



Cane et al., 1988

Shock Acceleration I

$$V_z \frac{\partial f_i}{\partial z} - \frac{\partial}{\partial z}\left(K_{i,zz} \frac{\partial f_i}{\partial z}\right) - \frac{1}{3}\frac{dV_z}{dz} v \frac{\partial f_i}{\partial v} = Q_i$$

$$\zeta(z) = \int_0^z [V/K_{zz}(z')] dz'$$

$$f(z>0,v) = f_0 - (f_0 - f_\infty)(1-e^{-\zeta})(1-e^{-\zeta_\infty})^{-1}$$

$$f_0 \equiv \beta \int_0^v \frac{dv'}{v'} f_\infty(v')(1-e^{-\zeta'_\infty})^{-1} \left(\frac{v}{v'}\right)^{-\beta} \exp\left[-\beta \int_{v'}^{v} \frac{dv''}{v''} e^{-\zeta''_\infty}(1-e^{-\zeta''_\infty})^{-1}\right]$$

Shock Acceleration II

$$-K_{zz}\frac{\partial f}{\partial z}\bigg|_{z\to\infty} = V(f_0 - f_\infty)e^{-\zeta_\infty}(1-e^{-\zeta_\infty})^{-1}$$

Shock Acceleration III

$$-V\partial I_{\pm} / \partial z = 2\gamma_{\pm} I_{\pm}$$

$$I \cong I_{+} = I_{+}^{\circ}(k) + \frac{4\pi^2}{k^2}\frac{V_A}{V}|\Omega_p| m_p \cos\psi \int_{|\Omega_p/k|}^{\infty} dv v^3 (1 - \frac{\Omega_p^2}{k^2 v^2})(f_p - f_{p,\infty})$$

$$f_{p,\infty} = \overline{n}_p (4\pi v_{p,0}^2)^{-1} \delta(v - v_{p,0}) + \overline{C} v^{-\gamma} S(v - \overline{v}_{p,0})$$

Shock Acceleration IV

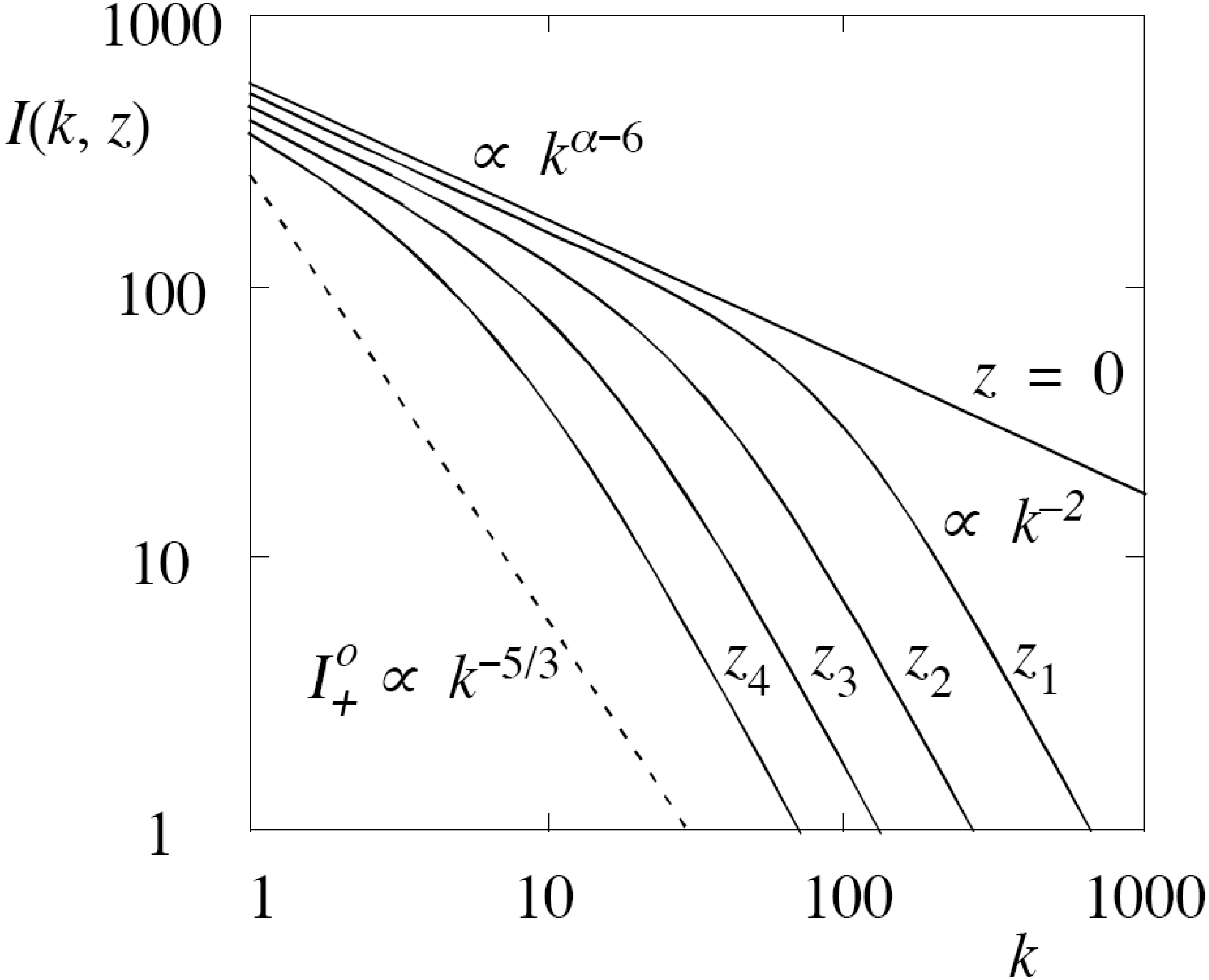
$$I = I_+^{\circ} + \frac{4\pi^2}{k^2}\frac{V_A}{V}|\Omega_p| m_p \cos\psi \int_{|\Omega_p/k|}^{\infty} dv v^3 (1 - \frac{\Omega_p^2}{k^2 v^2}) \cdot$$

$$\cdot \left\{ \frac{\beta \bar{n}}{4\pi v_{0,p}^3} (\frac{v}{v_{0,p}})^{-\beta} S(v - v_{0,p}) - \frac{\bar{n}}{4\pi v_{0,p}^2} \delta(v - v_{0,p}) \right.$$

$$\left. + \frac{\bar{C}\bar{v}_{0,p}^{-\gamma}}{\beta - \gamma} \left[\gamma (\frac{v}{\bar{v}_{0,p}})^{-\gamma} - \beta (\frac{v}{\bar{v}_{0,p}})^{-\beta} \right] S(v - \bar{v}_{0,p}) \right\} \cdot$$

$$\cdot \exp\left\{ -V \int_0^z dz' \left[\cos^2\psi \frac{v^3}{4\pi} \frac{B_0^2}{\Omega_p^2} \int_{-1}^{1} d\mu \frac{|\mu|(1-\mu^2)}{I(\Omega_p \mu^{-1} v^{-1})} + \sin^2\psi K_\perp \right]^{-1} \right\}$$

Shock Acceleration V



Analytical and Numerical Calculations Proposed I

1. Calculation of $I(k,z)$
   with dependence on $\psi, K_{\perp}, \xi_{p}(\psi), v_{0}(\psi)$

2. Role of shock obliquity (ψ)
   (Tylka and Lee, 2006)

Analytical and Numerical Calculations Proposed II

3. Form of exponential rollover
   with dependence on $E, \psi, A/Q, K_{\perp}$

4. Ion spatial profiles upstream of the shock
   with dependence on $E, \psi, A/Q, K_{\perp}$

5. Escaping ions and the streaming limit
   with dependence on $E, A/Q$

6. Inferring ξ, β from ESP events

$$I \propto k^{-\lambda} \qquad\qquad K_{\perp} = 0$$

Spectral Rollover:

$$\exp\left[-\beta \int_{v_0}^{v} \frac{dv'}{v'} \frac{e^{-\zeta_{\infty}^{'}}}{1-e^{-\zeta_{\infty}^{'}}}\right] \rightarrow \exp\left[-\beta \int_{v_0}^{v} \frac{dv'}{v'} \frac{1}{\zeta_{\infty}^{'}}\right]$$

$$\zeta_{\infty}^{i} \propto (Q_i / A_i)^{2-\lambda} v^{\lambda-3}$$

$$\rightarrow \exp\left[-C(\psi)(A_i / Q_i)^{2-\lambda} v^{3-\lambda}\right]$$

Mewaldt et al.; $E_0^i \propto (Q_i / A_i)^b$ $\qquad b = 2\dfrac{2-\lambda}{3-\lambda}$

Work Plan: Way Behind!

Year 1: Complete and publish analytical work

Year 2: Develop the numerical algorithm and
test the analytical approximation.

Year 3: Systematic comparison of results with
selected events